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نموذج رقم (١٨)
اقرار والتزام بالمعايير الأخلاقية والأمانة العلمية
وقوانين الجامعة الأردنية وأنظمتها وتعليماتها
لطلبة الماجستير

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عنوان الرسالة:

A Fuzzy Regression-based Approach for Optimizing the Multi-Response Problem in the Taguchi Method

اعلن بأنني قد التزمت بقوانين الجامعة الأردنية وأنظمتها وتعليماتها وقراراتها السارية المفعول المتعلقة باعداد رسائل الماجستير عندما قمت شخصيا" باعداد رسالتي وذلك بما ينسجم مع الأمانة العلمية وكافة المعايير الأخلاقية المتعارف عليها في كتابة الرسائل العلمية. كما أنني أعلن بأن رسالتي هذه غير منقولة أو مستلة من رسائل أو كتب أو أبحاث أو أي منشورات علمية تم نشرها أو تخزينها في أي وسيلة اعلامية، وتأسيسا" على ما تقدم فانني أتحمل المسؤولية بأنواعها كافة فيما لو تبين غير ذلك بما فيه حق مجلس العمداء في الجامعة الأردنية بالغاء قرار منحي الدرجة العلمية التي حصلت عليها وسحب شهادة التخرج مني بعد صدورها دون أن يكون لي أي حق في التظلم أو الاعتراض أو الطعن بأي صورة كانت في القرار الصادر عن مجلس العمداء بهذا الصدد.

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A FUZZY REGRESSION-BASED APPROACH FOR OPTIMIZING THE MULTI-RESPONSE PROBLEM IN THE TAGUCHI METHOD

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This Thesis was Submitted in Partial Fulfillment of the Requirements for theMaster's Degree of Science in Industrial Engineering/Management

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COMMITTEE DECISION

This Thesis (A Fuzzy Regression-based Approach for Optimizing the Multi-Response Problem in the Taguchi Method) was successfully defended and approved on May 5, 2011.

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DEDICATION

Firstly, I would like to dedicate my thesis to my deepest gratitude and utmost appreciation to my beloved parents; my father for his endless support and encouragement, who taught me that even the largest task can be accomplished if it is done one step at a time, and my mother for her infinite love and support which gave me the courage and patient to complete this work.

Secondly, this thesis would be incomplete without a mention of the support given me by my cherished cousin, Hashem, to whom this thesis is dedicated. He was my own "soul out of my soul," who kept my spirits up when the muses failed me. Without his lifting me up when this thesis seemed interminable, I doubt it should ever have been completed.

Thirdly, I would like to thank all my beloved brothers, and their lovely wives for their support to me, precious standing beside me, and for providing me a healthy atmosphere to work on this thesis. Many thanks for who stand beside me and treat me when I was sick. Not forgetting my lovely sisters, who stands beside me with their support and love, which gave me the courage to complete this work.

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My deep thanks go to my family for supporting me throughout all my studies, and my friends who were always there beside me.

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Reema Mohammad Abu Al-Haj

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Abbreviations

ANOVA	: Analysis of variance
ANN	: Artificial neural network
BOD	: Biochemical Oxygen Demand
DEA	: Data envelopment analysis
DR	: Deposition rate
ER	: Electrical resistively
EDM	: Electrical discharge machining
GAs	: Genetic Algorithms
GZO	: Gallium-doped zinc oxide
LH	: Left helix
LP	: Left profile
LTB	: Larger-the-better
MF	: Mechanical flotation
MO	: Microorganism
MRR	: Material removal rate
MRSN ratio	: Multi-response signal-to-noise ratio
NTB	: Nominal-the-best
OA	: Orthogonal array
OT	: Optical transmittance
PCA	: Principal component analysis
PLA	: Poly lactic acid
RB	: Real biodegradability
RH	: Right helix
RP	: Right profile

RSM	: Response Surface Methodology
S/N ratio	: Signal-to-noise ratio
SR	: Surface roughness
STB	: Smaller-the-better
WAS	: Waste activated sludge
WEDM	: Wire electrical discharge machining

Symbols

η	: The S/N ratio
K	: The number of replicates for response *y*
$\bar{y}$	: The estimated average of response *y*
s	: The estimated standard deviation
$\tilde{A}$	: A fuzzy set
X	: Universe of discourse
$\mu_{\tilde{A}}(x)$	: The grade membership function of x in $\tilde{A}$
l	: Lower limit
m	: Mean limit
u	:Upper limit
b	: *b* limit
c	: *c* limit
y_j	: The response variable
x_{fj}	: The independent factor variables for the *j* response and *f* factor
β_{fj}	: The coefficients (parameters) for the *j* response and *f* factor
ε	: Random error term
i	: The index of the experiment number, $i = 1, 2, \ldots, n$
j	: The index of the response number, $j = 1, 2, \ldots, q$
r	: The index of the replicate number, $r = 1, 2, \ldots, k$
f	: The index of the factor number, $f = 1, 2, \ldots, v$
l	: The index of the level number, $l = 1, 2, \ldots, m$
η_{ijr}	: The S/N ratio at each experiment *i* for each response *j*, and for each replicate *r*
y_{ir}	: The response *y* for replicate *r* at the *i*th experiment
$\bar{\eta}_{lfr}$	: The average of S/N ratio at level *l* of factor f for the *r*th replicate

η_{jr}	: The S/N ratio for response *j* and for *r*th replicate
$\tilde{\eta}_j$	: The fuzzy η for response *j*
$\tilde{\beta}$	: The fuzzy coefficient
β^l	: The lower limit of coefficient β
β^m	: The mean limit of coefficient β
β^u	: The upper limit of coefficient β
β^b	: The *b* limit of coefficient β
β^c	: The *c* limit of coefficient β
$\tilde{X}^*_{fj}$	: The optimal fuzzy factor levels for a factor f and for response *j*
$\tilde{\eta}_j(\tilde{X}^*_q)$	: The values of η for *j*th response by replacing the optimal fuzzy factor levels of *q*th response
$\tilde{X}^*_q$	: The optimal fuzzy factor levels of the *q*th response
$\tilde{d}_j(\tilde{\eta}_j(X))$	: The values of desirability function for *j*th response by replacing the values of $\tilde{\eta}_j(X)$
$\eta_{\max}$	: The maximum value of η
$\eta_{\min}$	: The minimum value of η
$\tilde{U}_j$	: The upper limit for the desirability function for response *j*
$\tilde{L}_j$	: The lower limit for the desirability function for response *j*
D_j	: The deviation function for *j*th response
$\tilde{Q}_j$	: The upper limit for the deviation function for response *j*
$\tilde{P}_j$	: The lower limit for the deviation function for response *j*
$\tilde{S}_j$	: The degrees of satisfaction from desirability function for response *j*
$\tilde{T}_j$	: The degrees of satisfaction from deviation function for response *j*
w_1	: The important weights for desirability
w_2	: The important weights for robustness

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ABSTRACT

The Taguchi method is a widely used technique for optimizing product/process parameter to improve products quality at a relatively low cost. Nevertheless, Taguchi method is only an effective technique for optimizing a single-response. In reality, most products/processes have several responses of main customer concern, and therefore, optimizing multi-response has become increasingly important. This thesis proposes an efficient approach for optimizing the multi-response problem in the Taguchi method utilizing fuzzy regression-based approach by applying fuzzy set theory. This approach proposes to constitute a regression model based on replicates of a response and aggregate regression models, so that a fuzzy regression model expresses each response. The obtained regression model includes triangular/trapezoidal fuzzy coefficients. Two previously-investigated case studies are provided for illustration. The computational results show that the proposed approach provides the largest satisfactory region. A validation experiment is then carried out to demonstrate the effectiveness of the proposed approach. In conclusion, the proposed approach could provide a great assistant to practitioners for optimizing the multi-responses in the manufacturing applications on the Taguchi method, if the data set is too small, or there is difficulty verifying that the error is normally distributed, and or if there is vagueness in the relationship between the process controllable factors and responses.

Chapter One
Introduction

This chapter introduces an overview of the Taguchi method, fuzzy sets, and fuzzy regression.

1.1. Taguchi method

The introduction of the robust design by (Taguchi, 1991), or the so-called Taguchi method, has become a powerful tool for producing high-quality products at low cost. The overall goal of robust design is to find settings of the controllable factors so that the response is least sensitive to variations in the noise variables, while still yielding an acceptable mean level of the response, where the noise variables may include parameters variations, environmental changes, operating conditions, and manufacturing variations. In this method (Belavendram, 1995), a fractional factorial designs, or the so-called orthogonal arrays (OAs), are usually utilized to reduce the number of experiments under permissive reliability. The columns of the OA represent process/product parameters to be studied, whereas the rows denote the individual experiments, where each row contains the combination of factor levels at which an experiment is conducted. The signal-to-noise (S/N) ratio is then employed to decide the optimal factor levels. The Taguchi method has been widely accepted for robust design in many business applications. Nevertheless, most of the published researches on the application of Taguchi method, such as (Yang, 2001), (Tsao and Hocheng, 2002), and (Li *et al*., 2008), have been conducted to optimize a single quality response of a process or product.

Typically, the quality response, y, is divided into three main types involving: (1) the smaller-the-better (STB) type response, in which the response is continuous,

nonnegative, and its most desired value is zero; (2) the nominal-the-best (NTB) type response, in which the response is continuous, nonnegative, and its target is nonzero and finite; and (3) the larger-the-better (LTB) type response, which can be transformed into the STB type by considering the reciprocal of the response. Then, the S/N ratio is employed to measure performance. Regardless of the response type, the optimal factor level is the factor level which maximizes the S/N ratio, η. That is, the objective function to be maximized is:

$$\eta = \begin{cases} -10\log_{10}\left(\frac{1}{K}\sum_{r=1}^{K}(y_r^2)\right) & \text{for STB} \\ 10\log_{10}(s^2/\bar{y}^2) & \text{for NTB,} \\ -10\log_{10}\left(\frac{1}{K}\sum_{r=1}^{K}(1/y_r^2)\right) & \text{for LTB} \end{cases} \tag{1.1}$$

where K denotes the number of replicates for response y. The $\bar{y}$ and s_i are the estimated average and standard deviation, respectively.

1.2. Fuzzy Sets and Membership Function

Fuzzy sets introduced by (Zadeh, 1965) were used to represent and manipulate data and information which possess non-statistical uncertainties. It was designed to mathematically represent uncertainty and vagueness, and to provide formalized tools for dealing with the imprecision intrinsic to many problems differentiated by the membership functions.

A membership function (Zadeh, 1965) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between zero and one. The simplest membership functions are formed using straight lines. Examples include triangular membership function and trapezoidal membership function.

A fuzzy set $\tilde{A}$ in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$, which associates with each element x in X a real number in the interval [0,1]. The function value of $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in $\tilde{A}$ (Zimmermann, 1987).

A triangular fuzzy number $\tilde{A}$,as shown in Figure 1.1, can be defined by a triplet (l,m,u), where l , m, and u, are the lower, mean, and upper limits, respectively.

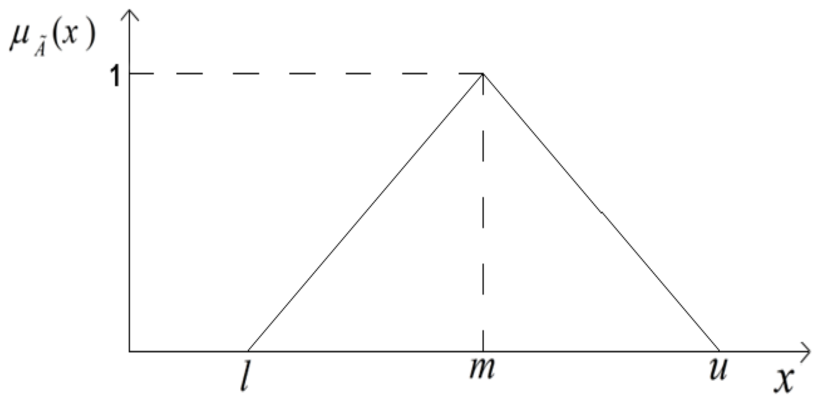


Figure 1.1: Triangular membership function.

The $\mu_{\tilde{A}}(x)$ is then defined as:

$$\mu_{\tilde{A}}(x)=\begin{cases} 0, & x \le l \\ \dfrac{x-l}{m-l}, & l \le x \le m \\ \dfrac{u-x}{u-m}, & m \le x \le u \\ 0, & x \ge u \end{cases} \tag{1.2}$$

Let $\tilde{A}=[a_1,a_2,a_3]$ and $\tilde{B}=[b_1,b_2,b_3]$ two triangular fuzzy numbers then the arithmetic operations on $\tilde{A}$ and $\tilde{B}$ as follows:

$$\tilde{A}+\tilde{B}=(a_1+b_1,a_2+b_2,a_3+b_3) \tag{1.3.a}$$

$$\tilde{A} - \tilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1) \tag{1.3.b}$$

$$\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3) \tag{1.3.c}$$

A trapezoidal fuzzy number $\tilde{B}$ as shown in Figure 1.2 can be defined as (l, b, c, u), where l , b, c, and u, are trapezoidal limits.

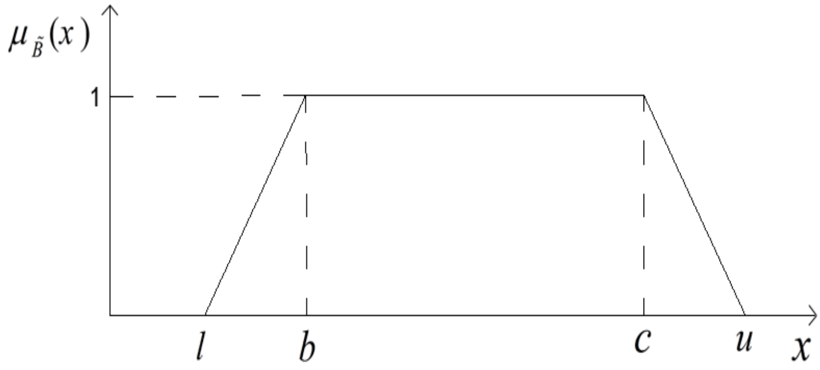


Figure 1.2: Trapezoidal membership function.

The $\mu_{\tilde{B}}(x)$ is then defined as:

$$\mu_{\tilde{B}}(x) = \begin{cases} \dfrac{x-l}{b-l}, & l \le x \le b \\ 1, & b \le x \le c \\ \dfrac{x-u}{c-u}, & c \le x \le u \\ 0, & x \le l, x \ge u \end{cases} \tag{1.4}$$

Let $\tilde{A} = [a_1, a_2, a_3, a_4]$ and $\tilde{B} = [b_1, b_2, b_3, b_4]$ two trapezoidal fuzzy numbers then the arithmetic operations (Zimmermann, 1987) on $\tilde{A}$ and $\tilde{B}$ as follows:

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \tag{1.5.a}$$

$$\tilde{A} - \tilde{B} = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1) \tag{1.5.b}$$

$$\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4) \tag{1.5.c}$$

1.3. Fuzzy Linear Regression

Statistical regression is a comprehensive and powerful tool that can be used in finding a crisp relationship between the dependent and independent variables and also used to estimate the variance of measurement error.

Standard statistical multiple linear regressions take the form (Montgomery, 2010)

$$y = \beta_o + \beta_1 x_1 + ... + \beta_f x_f + \varepsilon, \forall f \tag{1.6}$$

where, y is the response variable, $x_1,...,x_f$ are the independent variables and the coefficients, β_f , are crisp values, and ε is a random error term.

Although statistical regression has many applications; (Shu *et al*., (2006); Hasan *et al*., (2006); and Gopalsamy *et al*., (2009)), problems can occur in the following situations:

i. Number of observations is inadequate (small data set).

ii. Difficulties verifying distribution assumptions.

iii. Vagueness in the relationship between input and output variables.

iv. Ambiguity of events or degree to which they occur.

v. Inaccuracy and distortion introduced by linearization.

The fuzzy regression analysis uses fuzzy numbers, which can be expressed as intervals with membership values as the regression coefficients, was first introduced by (Tanaka, *et al*., 1982). Since the regression coefficients are fuzzy numbers, the estimated dependent variable will be a fuzzy number.

Fuzzy linear regressions take the form (Tanaka, *et al*., 1982)

$$\tilde{y} = \tilde{\beta}_o + \tilde{\beta}_1 x_1 + ... + \tilde{\beta}_f x_f , \forall f \tag{1.7}$$

where $\tilde{y}$ is the fuzzy output, the independent variables, x_f , and the coefficients, $\tilde{\beta}_f$, are fuzzy values.

Thus, fuzzy regression is efficient if the data set is too small, or there is difficulty verifying that the error is normally distributed, or if there is vagueness in the relationship between the independent and dependent variables, or if there is ambiguity associated with the event or if the linearity assumption is inappropriate. These are the situations fuzzy regression was meant to address.

1.4. Research Problem Statement

The optimization of multi-response has received increasing attention over the last few years in many manufacturing organizations, as well as academic research (Antony, (2001); Liao and Chen, (2002); Liao, (2005); and Al-Refaie *et al*., (2008)). The Taguchi method of experimental design is a widely accepted technique for producing high quality products at low cost. It is observed that most published research (Ghani *et al*., (2004); Chih *et al*., (2005); Mishra *et al*.,(2009); and Chang, (2010)) on the Taguchi have been primarily concerned with the optimization of a single response. Many manufacturing processes tend to be very complex in behavior and have inherent system fuzziness such as fluctuation of process pressure and temperature due to environmental effects. Fuzzy regression has been found to be more appealing than statistical regression in estimating the relationship between the dependent variable and independent variables when a high degree of fuzziness is involved and only a few data sets are available for model building. Therefore, this thesis proposes an efficient approach for optimizing the multi-response problem in the manufacturing application on the Taguchi method by fuzzy regression-based approach.

1.5. Research Objectives

The presented approach will propose an effective optimization approach for multi-responses based on the Taguchi method and fuzzy regression-based approach, which has the following benefits:

i) Optimizing mean and variance simultaneously without setting mean in a distinct value or range.

ii) Determining appropriate levels of design parameters so that the responses become robust against noise; such as manufacturing variations and environmental disturbances.

iii) Applying fuzzy concepts as a tool for considering mean and variance considering uncertainty of data collected, showing factor levels as a fuzzy number with a capability to change them at an acceptable range based on experimental conditions.

iv) Taking into consideration decision making preferences for optimizing mean or variance by assigning importance weight to them.

1.6. Thesis overview

The remaining of this thesis is organized as follows. Chapter Two presents several related researches. Chapter Three outlines the proposed approach. Chapter Four illustrates the proposed approach by considering two case studies in manufacturing applications, which were investigated in previous literature, and a validation experiment is carried out. Chapter Five summarizes the research results. Chapter Six provides the conclusions and future research.

Chapter Two

Literature Review

This chapter presents several related previous research in the application on the Taguchi method for optimizing single-response and multi-response problems.

2.1. Previous research on optimizing single-response problem in the Taguchi method

2.1.1. Signal-to-noise ratio

Ghani, *et al*. (2004) applied the Taguchi optimization methodology to optimize cutting parameters in end milling to find a combination of milling parameters that optimize two STB type responses separately: cutting force and surface roughness. The milling parameters evaluated, including: cutting speed, feed rate and depth of cut. The L_{27} array, S/N ratio and analysis of variance (ANOVA) are employed to analyze the effect of these milling parameters.

Chih, *et al*. (2005) presented the optimization of electroplating conditions of nickel-coated biochips, where the Taguchi method was adopted to determine the optimal process immune assay, LTB type response of the nickel-coated chip using L_{18} array to investigate eight electroplating parameters, including: electroplating buffer, four kinds of electroplating additives, current density, and temperature of electroplating buffer. Then, the immune assay was utilized to investigate the properties of the protein chip.

Shing and Hwee (2005) investigated micro parts having a dimension in the order of micron scale. Parameters for the filling of polymers into a thin cavity were studied. Subsequently, the thickness of the cavity was progressively reduced and the percent

contribution of each factor was obtained. A comparison of a completely filled and partially filled case was presented with the aid of the Taguchi technique. The L_9 array was employed to investigate the effect of four process factors; namely, injection rate, injection pressure, mold temperature, and injection time, on LTB type response; namely, average part weight,

Barend and Villiers (2007) evaluated the potential of mechanical flotation (MF) to thicken waste activated sludge (WAS) as a percentage and are referred to as the thickening ratio (TR). Taguchi's L_{16} array was employed to investigate the effect of five process factors, including: airflow rate, residence time, impeller speed, surfactant dosing, and sludge dilution. The objective was to determine the potential of MF to thicken WAS by using the Taguchi experimental design method.

Mishra, *et al*. (2009) studied erosion wear behavior of fly ash–illmenite coating using dry silica sand as an erodent. To obtain optimal parameters, the LTB type response for the erosion wear rate utilizing L_9 array, and S/N ratio was employed to investigate the influence of four factors including: the impact angle, the velocity, the size of the erodent, and the standoff distance on erosion wear. The objective was to investigate which design parameter significantly affects the erosion wear.

Tavakoli, *et al*. (2009) investigated the effect of five operating parameters; namely, decomposition temperature, bubbler temperature, carrier gas flow rate, precursor flow rate and pressure on particle size in the chemical vapor condensation process using L_8 array, ANOVA was employed to determine the significant parameters.

Xiaoming, *et al*. (2009) studied the influence of forging parameters on microstructure uniformity and smallness in order to improve quality of heavy forgings. Four factor process using L_9 array; namely, ingot height to diameter ratio, the scope of

initial forging temperature, the top die velocity, and total reduction, that optimize microstructure, which is STB type response.

Chang (2010) optimized the biodegradability of poly lactic acid (PLA). The L_8 array was employed to investigate the influence of four factors, including: temperature, bacteria, ventilation degree, and nutrient, on initial-stage hydrolysis rate of PLA, which is LTB type response.

2.1.2 Regression analysis

Shu, *et al*. (2006) presented the use of the Taguchi-regression method in developing the optimal plasma transferred arc welding process for obtaining high hardfacing quality responses. The L_{18} array was employed to investigate the influence of eight factor processes; namely, hardfacing material, accelerating voltage, electrical current, powder feed rate, waving oscillation, plasma gas rate, rotation, and pre-heat treatment, on the wear weight loss, which is the STB type response.

Hasan, *et al*. (2006) developed a Taguchi-regression optimization method for surface roughness in finish milling of mold surfaces. Considering the process parameters of feed, cutting speed, axial-radial depth of cut, and machining tolerance. The L_{18} array was used to measure the surface roughness, which is STB type response.

2.1.3 Fuzzy Regression approach

Xue, *et al*. (2005) developed a fuzzy linear regression technology in the Taguchi method to study the relationship between four process variables; i.e., wire diameter, arc voltage, welding speed and welding current and bead width, which is STB type response of the robotic arc-welding process using L_{16} array. The developed model can fit variables to a linear function which a process of determining coefficients of the

fitting function to study the quality of the welding and determine process variables for obtaining the optimal bead width.

Table 2.1 summarizes the previous research on optimizing single-response problem in the Taguchi method.

2.2. Previous research on optimizing multi-response problem in the Taguchi method

2.2.1 Multi-response signal-to-noise ratio approach

In multi-response S/N ratio (MRSN) approach, the Taguchi method is applied towards a multi-response production process. Considering the quality loss of each response, a multi-response S/N ratio is developed to determine the optimum conditions in the parameter design stage for the multi-response problem.

Phadke (1989) optimized multi-response individually. Then, a trade-off between quality loss and productivity by engineering judgment was used to decide the optimal factor levels. Unfortunately, this approach increases uncertainty in the decision-making process.

Antony (2001) presented MRSN ratio to optimize three quality responses; namely, solder paste mass (NTB response), solder paste height (NTB response), and glue torque (LTB response) utilizing L_8 array to investigate the effect of six controllable factors.

Lin (2002) studied MRSN ratio for optimizing face milling stainless steel process utilizing L_9 array to investigate the effect of three cutting parameters; namely, cutting speed, feed rate, and depth of cut, on the three responses; namely, removed volume (LTB response), surface roughness (STB response), and burr height (STB response).

Ramakrishnan and Karunamoorthy (2006) performed MRSN ratio to optimize three responses; namely, material removal rate (LTB response), surface roughness (STB

response), and wire wear ratio (STB response) of wire electrical discharge machining (WEDM) operations utilizing L_{16} array to investigate the effect of five cutting parameters, including, pulse on time, wire tension, delay time, wire feed speed, and ignition current intensity.

Ramakrishnan and Karunamoorthy (2008) described the development of artificial neural network (ANN) models and MRSN ratio to predict and select the best cutting parameters of WEDM process. The L_9 array was employed to investigate the effect of four cutting parameters; namely, pulse on time, delay time, wire feed speed, and ignition current, on the two responses; namely, material removal rate (LTB response) and surface roughness (STB response).

Al-Refaie, *et al*. (2010) proposed an approach for solving the multi-response problem in the Taguchi method. Each quality response transformed into S/N ratio. The average S/N ratio calculated for each factor level, and then weighted with respect to the level of the largest average S/N ratio for this factor. The average weight of each factor level, or level weight, obtained from all responses. The factor level with the largest level weight was selected as the optimal level for that factor. Three case studies in manufacturing were employed for illustration of their proposed approach, one of them was optimizing of a gear hobbing operation, L_{18} array was employed, and four STB type quality responses were selected; namely, left profile (LP) error, right profile (RP) error, left helix (LH) error, and right helix (RH) error. Six process controllable factors were investigated, including: direction of hobbing, number of passes, source of hob, feed, speed, and job run out.

2.2.2 Data envelopment analysis based ranking approach

A data envelopment analysis (DEA) based ranking approach is proposed to optimize effectively the multi-response problem in the Taguchi method.

Liao and Chen (2002) proposed a DEA based ranking approach for optimizing the multi-response problem in Taguchi method. Two case studies, which were investigated in previous literature, were provided for illustration, one of them was optimizing of polysilicon process, and the Taguchi method was used to improve the quality of polysilicon process by optimizing concurrently three responses; the surface defects (STB response), thickness (NTB response) and deposition rate (LTB response) utilizing L_{18} array. Six process factors were investigated simultaneously including: deposition temperature, deposition pressure, Nitrogen flow, silane flow, settling time, and cleaning method.

Liao (2005) used ANN and DEA to achieve the optimization of multi-response problem. Two case studies, which were investigated in previous literature, were provided for illustration, one of them was improving the quality of hard disk drive. The L_{18} array was utilized to investigate the effect of five controllable factors, including: disk writability, magnetization width, gap length, coercivity of media, and rotational speed, and one uncontrollable factor: flying height (noise). The four desired responses were: 50% pulse width (STB response), peak shift (STB response), high-frequency amplitude (LTB, type response), and over write (LTB response).

Al-Refaie and Li (2008) proposed an approach for solving the multi-response problem in Taguchi method by aggressive formulation in DEA. Two case studies, which were investigated in previous literature, were provided for illustration, one of them was optimizing of polysilicon process, and the Taguchi method was used to improve the quality of polysilicon process by optimizing concurrently three responses; the surface

defects (STB response), thickness (NTB response) and deposition rate (LTB response) utilizing L_{18} array. Six process factors were investigated simultaneously including: deposition temperature, deposition pressure, Nitrogen flow, silane flow, settling time, and cleaning method.

Al-Refaie, et al. (2009) proposed a procedure for solving the multi-response problem in the Taguchi method utilizing two DEA approaches, including comparisons of efficiency between different systems and bilateral comparisons. Three case studies in manufacturing were employed for illustration of the proposed approach; one of them was optimizing of hard disk drive. Four responses were chosen, including 50% pulse width (STB response), peak shift (STB response), over write (LTB response), and high-frequency amplitude (LTB response) utilizing L_{18} array. Five controllable factors were investigated involving disk writability, magnetization width, gap length, coercivity of media, and rotational speed.

2.2.3 Grey relational analysis and desirability function approach

In the grey relational analysis method (Deng, 1982), experimental data are first normalized in the range between zero and one, which is also called the grey relational generation. Next, the grey relational coefficient is calculated from the normalized experimental data to express the relationship between the desired and actual experimental data. Then, the grey relational grade is computed by averaging the grey relational coefficient corresponding to each process response. The overall evaluation of the multiple process responses is based on the grey relational grade. The optimal level of the process parameters is the level with the highest grey relational grade. For examples:

Lin (2004) used of the taguchi method and grey relational analysis to optimize turning operations with multi responses. Three responses were optimized; tool life (LTB response), cutting force and surface roughness (STB responses). Three process controllable factors were investigated, including: cutting speed, feed rate, and depth of cut utilizing L_9 array.

Palanikumar, *et al*. (2006) focused on optimization of multiple machining responses of glass fiber–reinforced polymer composites produced through filament winding using carbide tool. The L_{27} array was utilized to investigate the effect of five machining parameters, including: work piece fiber orientation, cutting speed, feed rate, depth of cut, and machining time, on the multi-responses, including: material removal rate (LTB response) and (STB responses) for tool wear, surface roughness and specific cutting pressure.

Pan, *et al*. (2007) presented optimization of multi-responses of Nd: YAG laser welded titanium alloy plates via Taguchi method-based Grey analysis. The L_{18} array was employed. The essential machining parameters were identified as: shielding gas, laser energy, traveling speed, laser focus, pulse frequency, and pulse shape. Furthermore, the multi-responses required included: ultimate tension stress (LTB response), heat-affected zone (STB response), welding depth to width ratio (LTB response), and surface roughness (STB response).

Kopac and Krajnik (2007) utilized the application of grey-Taguchi method to optimize flank milling parameters. The multi-response including: surface roughness and resultant force were STB responses, whilst material removal rate was LTB response. Six controllable flank milling parameters were investigated, including: coolant application, number of flutes, cutting speed, feed per tooth, axial depth of cut, and radial depth of cut, utilizing L_{16} array.

Datta, *et al*. (2008) optimized features of bead geometry using grey relational analysis. Six responses were optimized, including; the four STB responses; namely, weld penetration shape factor, weld reinforcement form factor, area of reinforcement, and total bead cross sectional area, and also the two LTB responses; namely, area of penetration and percentage dilution. Four process controllable factors were investigated, including: current, slag-mix %, and basisity index, utilizing L_{18} array.

Al-Refaie, *et al*. (2008) optimized stainless steel 304 (SUS 304) wire drawing process with two responses; the die life and wire tensile strength, which were both the LTB responses. Three control factors were investigated, involving: reduction ratio, lubricant temperature, and drawing speed utilizing L_9 array.

Lin and Lee (2009) developed a novel process of magnetic- force-assisted electrical discharge machining (EDM) and conducted an experimental investigation to optimize the machining parameters associated with multi-response using gray relational analysis. The L_{18} array was employed to investigate the effect of six machining parameters, including: machining polarity, peak current, pulse duration, high-voltage auxiliary current, no-load voltage, and servo reference voltage, on three responses; namely, the material removal rate (LTB response), electrode wear rate (STB response), and surface roughness (STB response).

Muruganantham, *et al*. (2009) determined the optimal settings of the parameters that will maximize the performance of the machine vision system using desirability function approach. The L_{27} array was used to investigate the effect of four vision parameters; namely, illumination intensity, length of extender tube, number of control points, and region of interest, were optimized on multi-response; namely, accuracy and repeatability of calibration and distorted image plane error. The objective is to

determine the best factor settings, which will simultaneously optimize all multi-response.

Chen, *et al*. (2010) examined the optimization of the process parameters of gallium-doped zinc oxide (GZO) films deposited on polyethylene terephthalate substrates by R.F. magnetron sputtering using the Taguchi method, aiming to obtain highly transparent and conductive films. The L_{18} array was adopted to investigate the influences of the various sputtering factors, including: R.F. power, sputtering pressure, deposition time, substrate temperature and post-annealing temperature, on deposition rate (LTB response), electrical resistively (STB response), and structural, morphological and optical transmittance (LTB response) of GZO films.

Nandi, *et al*. (2010) adopted two hybrid techniques, the Taguchi method coupled with grey relational analysis, and the Taguchi method in combination with desirability function approach to search an optimal process environment, capable of producing desired high quality submerged arc weldment. Four bead geometry parameters: reinforcement, and bead width, which were STB responses, while depth of penetration, and percentage dilution, which were LTB responses. Four process parameters, including: voltage, wire feed rate, traverse speed and electrode stick-out were investigated utilizing L_{25} array. .

2.2.4 Principal Component Analysis

Principal component analysis (PCA) involves a mathematical procedure that transforms a set of correlated response variables into a smaller set of uncorrelated variables called principal components (Hotelling, 1933). For examples, Antony (2000) proposed a methodology based on Taguchi's parameter design methodology utilizing Taguchi's quality loss function and PCA. The optimization of a submerged arc-welding

process was used to verifying the proposed approach. The responses of interest were deposition rate (LTB response) and dilution (STB response). Five factors, arc current, arc voltage, welding speed, electrode protrusion, and preheat temperature were investigated utilizing L_8 array.

Tong (2005) proposed a novel optimization procedure for multi-response based on Taguchi's parameter design and PCA to obtain a set of uncorrelated components. A case study involving optimization of the chemical-mechanical polishing of copper thin films from an integrated circuit manufacturer was presented. Three responses were optimized; removal rate (LTB response), non-uniformity (STB response), and TaN/Cu selectivity (LTB response). Five control factors were investigated utilizing L_{18} array.

Fung and Kang (2005) optimized the injection-molding process for friction properties of fiber-reinforced polybutylene terephthalate using Taguchi method and PCA. The L_9 was conducted with four controllable factors of the manufacturing process, including: the filling time, melt temperature, mold temperature, and ram speed. The two STB responses were the friction coefficients and surface roughness in different sliding directions.

Huang and Lin (2008) proposed a methodology for setting the robust process parameters for injection molding based on PCA and a regression model-based searching method. The required responses, they contained (STB responses) of maximal shear stress, maximal warpage, and maximal volumetric shrinkage rate. The L_{18} array was employed to investigate the effect of eight factors, including: cooling time, plastic temperature, mold temperature, filling speed, holding pressure, holding time, injection pressure, and screw stroke.

Gauri and Chakraborty (2009) optimized the WEDM process using PCA. The aim of this study was to optimize the settings of six controllable factors, e.g., pulse on time,

pulse off time, peak current, wire tension, servo reference voltage, and dielectric flow rate, with respect to three performance characteristics of a WEDM process. Three responses were considered as cutting speed (LTB response), surface roughness (STB response) and dimensional deviation (STB response). For all the control factors, L_{18} was utilized.

2.2.5Fuzzy logic approach

Fuzzy logic analysis, including the max–min fuzzy inference and centroid defuzzification methods (Zadeh, 1965), is applied to deal with multi-response using the fuzziness of human concepts. The fuzzy-based Taguchi method can also be used to optimize the multi-response process through the settings of process parameters (Lin Wang, *et al*. 2000). As a result, the complicated interrelationships among the multi-response can also be solved by the fuzzy-based approach.

Tarng, *et al*. (2000) used of fuzzy logic in the Taguchi method to optimize the submerged arc welding process with multi-response. The L_8 array was employed. The process parameters were arc current, arc voltage, welding speed, electrode protrusion, and preheat temperature with considerations of the multi-response, including deposition rate (LTB response) and dilution (STB response).

Palanikumar (2006) optimized of machining parameters in turning glass fiber reinforced polymer composites using carbide tool based on the taguchi method with fuzzy logics. The L_{27} array was employed. The machining parameters, including: work piece (fiber orientation), cutting speed, feed rate, depth of cut and machining time were optimized with consideration of multi-response; namely, metal removal rate (LTB response), tool wear (STB response), and surface roughness (STB response).

Wang and Jean (2006) optimized the cobalt-based hardfacing in carbon steel using the fuzzy analysis for the robust design. Two responses were optimized; namely, vicker hardness (LTB response) and dilution percentage (NTB response). The eight controllable factors were metal powder, powder feed rate, electrical current, preheat temperature, oscillation speed, travel speed, pilot gas flow rate, and welding torch stand-off, were investigated utilizing L_{18} array.

Further, PCA was integrated with other approaches; such as:

Fang, *et al*. (2008) integrated a PCA with fuzzy theory for the extended optimization of multi-response in optimization experiments of non-image optics; a miniature optical engine. L_{18} array was employed, and three responses to be measured were the luminous flux (LTB response), illumination uniformity performance (LTB response), and the packing size (STB response). Eight process controllable factors were investigated, including: lens specifications, lens length, semi-aperture, integrator length, integrator width, TIR prism entering semi-diameter, TIR prism air gap, and DMD location. Lin and Lin (2005) used of the grey-fuzzy logic for optimizing the electrical discharge machining process with multi-response. The machining parameters, including: pulse on time, duty factor, and discharge current with considerations of multi-response; namely, electrode wear ratio (STB response), material removal rate (LTB response), and surface roughness (STB response) using L_9 array. Moreover, Chang and Lu (2007) presented an optimal cutting-parameter design of heavy cutting in side milling for SUS304 stainless steel. The OA with grey-fuzzy logics is applied to optimize the side milling process with multi-responses. The selected cutting parameters were spindle speed, feed per tooth, axial depth of cut and radial depth of cut, while the considered multi-response were tool life and metal removal rate with LTB responses, and L_9 array was employed.

Liu, *et al*. (2009) proposed a method of grey-fuzzy logic to achieve the optimization of multi-response during the manufacturing process. The light guide plate printing process was used for verification. The L_8 array was employed, the printing processing parameters; namely, mixed rate of ink, velocity and pressure of printing process, and material and angle of scraper, with considerations to the multiple machining responses, including illumination and homogeny were regarded as the LTB responses, where value of variance for the illumination was the STB response, and printing ink thickness was the NTB response.

2.2.6Multiple regression-based integration approach

Gopalsamy, *et al*. (2009) applied Taguchi method with regression analysis to find optimum process parameters for end milling while hard machining of hardened steel. L_{18} array, S/N ratio were applied to study performance characteristics of machining parameters, including: cutting speed, feed depth of cut, and width of cut, with consideration of surface finish (STB response) and tool life (LTB response). Multiple regression equations were formulated for estimating predicted values of two responses.

Pal and Gauri (2010) examined the effectiveness of the various performance metrics for multi-response optimization using multiple regressions. Three case studies were taken from literature. One of them was improving a polysilicon deposition process. Six controllable factors were identified: deposition temperature, deposition pressure, nitrogen flow, silane flow, setting time and cleaning method. L_{18} array was used. Three responses were considered; namely, surface defects (STB response), thickness (NTB response), and deposition rate (LTB response).

Table 2.2 summarizes the previous research on optimizing multi-response problems in the Taguchi method.

In the Taguchi method, to solve the multi-response problem, engineering judgment is the primary method. But, without doubt, the judgment of an engineer will increase uncertainty during the decision-making process. In order to solve the multi-response problem, an approach for assigning weight to each response is also used. However, it still remains difficult to determine and define a weight for each response in a real case. Another method is the regression technique based approach. However, such an approach is problematic if the data is too small, or if there is vagueness in the relationship between the independent and dependent variables, and the possible correlations among the responses may still not be considered. Furthermore, the significant factor in a single response case may not be considered significant when considered in a multi-response case. Therefore, in order to solve this multi-response problem, PCA method is considered. However, PCA has two shortcomings: (1) when multiple principal components of an eigenvalue that is greater than one are chosen, how to trade off to decide the feasible solution is still unknown; and (2) when the selected principal components have less variation than can be explained by total variation, the performance index of multi-responses is not evident enough to replace the original response variables. Another approach is DEA method, unfortunately, most of the traditional DEA models allow for complete weight flexibility which may result in identifying a decision making unit with an unrealistic weighing scheme.

2.3 Fuzzy regression approach

Fuzzy linear regression approaches have been successfully applied to some problems such as cost estimation of wastewater treatment (Wen and Lee, 1999), financial forecasting (Oh *et al*., 1990), earthquake prediction (Bardossy *et al*., 1992),

sales volume estimation (Watada, 1992), quality function deployment (Chen *et al*., 2004) and ergonomics (Chang *et al*., 1996).

Nevertheless, few research regarding process optimization have been attempted in previous research such as response surface methodology (RSM); Bashiri and Hosseininezhad (2009) proposed a methodology for optimizing multi-response surface in robust designs using fuzzy programming, assumptions were used. Three factors at two levels were studied using full factorial design; genetic algorithms (GAs) and fuzzy optimization. RSM is the most common technique for process optimization and has been applied successfully in various manufacturing processes such as electroplating and transfer moulding. Xie and Lee (1994) developed a fuzzy logic, RSM-based process optimization algorithm with CVD process as an application. The constructed fuzzy logic model was employed to evaluate the gradient and the gradient descent method was then used to optimize the objective function. GAs have been applied successfully to the optimization of various manufacturing processes, such as injection moulding and grinding. Tan and Yuen (2000) attempted to treat the optimization of process parameters setting for injection moulding as a multi-objective optimization using the fuzzy goal programming approach. The requirements of minimizing the objective functions and satisfying the constraints were expressed as a set of fuzzy goals with a linear type of membership function.

Therefore, and in light of the above context, this thesis proposes an efficient approach for solving the multi-response problem in manufacturing application on the Taguchi method and fuzzy regression based approach, Fuzzy linear regression otherwise provides a means for tackling regression problems lacking a significant amount of data for determinant regression models and with vague relationships between

the dependent variable and independent variable, which considers uncertainty of the collected data, so it leads to a better decision about the optimal regions.

Table 2.1: Summary of the previous research on optimizing single-response problem in the Taguchi method.

Paper	Objective	Process	Quality Characteristics	OA	Optimization Approach
Ghani, *et al*. (2004)	Optimizing cutting parameters in end milling.	End milling	Cutting force, and surface roughness	L_{27}	S/N ratio
Chih, *et al*. (2005)	Optimization of electroplating conditions.	Nickel-coated biochips	Immune assay	L_{18}	S/N ratio
Shing and Hwee (2005)	To investigate micro parts having a dimension in the order of micron scale.	injection molding machine	Average part weight	L_9	S/N ratio
Barend and Villiers (2007)	To evaluate the potential of (MF) to thicken (WAS).	WAS and MF	Thickening ratio	L_{16}	S/N ratio
Mishra, *et al*. (2009)	To investigate which design parameter significantly affects the erosion wear.	Fly ash–illmenite coating	Erosion wear rate	L_9	S/N ratio
Tavakoli, *et al*. (2009)	To investigate the effect of various operating parameters on particle size.	chemical vapor condensation	Particle size	L_8	S/N ratio
Xiaoming, *et al*. (2009)	Investigate the influence of parameters on microstructure uniformity.	Heavy forgings	Microstructure	L_9	S/N ratio

Table 2.1: Continued

Paper	Objective	Process	Quality Characteristics	OA	Optimization Approach
Chang (2010)	To optimize the Biodegradability of PLA.	PLA	Initial-stage hydrolysis rate	L_8	S/N ratio
Shu, *et al*. (2006)	To develop the optimal plasma transferred arc welding process	Plasma transferred arc welding process	Wear weight loss	L_{18}	Taguchi-regression method
Hasan, *et al*. (2006)	To optimize surface roughness in finish milling of mold surfaces	Finish milling of mold surfaces	surface roughness	L_{18}	Taguchi-regression method
Xue, *et al*. (2005)	To study the relationship between four process variables and a single response	Robotic arc-welding	Bead width	L_{16}	Fuzzy regression approach

Table 2.2: Summary of the previous research on optimizing multi-response problem in the Taguchi method.

Paper	Objective	Process	Quality Characteristics	OA	Optimization Approach
Antony (2001)	To determine the best settings of the process parameters, thereby making the process functional performance insensitive to various sources of variation.	Double-sided surface mount technology electronic	Solder paste mass, solder paste height, and glue torque	L_8	MRSN ratio
Lin (2002)	To improve the multi- response characteristics, and the weighting factor that significantly affect them	Face milling stainless steel	Removed volume, Surface roughness, and Burr height	L_9	MRSN ratio
Ramakrishnan and Karunamoorthy (2006)	To optimize three responses of cutting parameters of WEDM process.	WEDM	Material removal rate and surface roughness, and wire wear ratio	L_{16}	MRSN ratio
Ramakrishnan and Karunamoorthy (2008)	To predict and select the best cutting parameters of WEDM process.	WEDM	Material removal rate and surface roughness	L_9	ANN and MRSN ratio
Al-Refaie, *et al*. (2010)	An approach for solving the multi-response problem in the Taguchi method.	Gear Hobbing Operation	LP, RP, LH, and RH	L_{18}	Weighted S/N ratio
Liao and Chen (2002)	To optimize the multi-response problem in Taguchi method	Polysilicon process	surface defects, thickness, and deposition rate	L_{18}	DEA approach

Table 2.2: Continued

Paper	Objective	Process	Quality Characteristics	OA	Optimization Approach
Liao (2005)	To achieve the optimization of multi-response problem.	Hard disk drive	50% pulse width, peak shift, high-frequency amplitude, and over write	L_{18}	ANN and DEA
Al-Refaie and Li, (2008)	Proposed an approach for solving the multi-response problem in Taguchi method by aggressive formulation in data envelopment analysis	Polysilicon process	The surface defects, thickness, and deposition rate	L_{18}	DEA approach
Al-Refaie, et al. (2009)	Comparisons of efficiency between different systems and bilateral comparisons	Hard disk drive	50% pulse width, peak shift, high-frequency amplitude, and over write	L_{18}	DEA approach
Lin (2004)	To optimize turning operations with multiple responses	Turning operations	Tool life, cutting force and surface roughness	L_9	Grey Relational Analysis
Palanikumar, *et al*. (2006)	To optimize the multiple performance machining characteristics of glass fiber–reinforced polymer composites produced through filament winding	Glass fiber–reinforced polymer	Material removal rate, and tool wear, surface roughness	L_{27}	Grey Relational Analysis
Pan, *et al*. (2007)	To optimize of multiple quality characteristics of Nd:YAG laser welded titanium alloy plates.	Laser welded titanium alloy plates	Tension stress, heat-affected zone, welding depth to width ratio, and surface roughness.	L_{18}	Grey Relational Analysis
Kopac and Krajnik (2007)	To optimize flank milling parameters.	Flank milling	surface roughness, resultant force, and material removal rate	L_{16}	Grey Relational Analysis

Table 2.2: Continued

Paper	Objective	Process	Quality Characteristics	OA	Optimization Approach
Datta, *et al*. (2008)	To optimize features of bead geometry.	Bead geometry	Weld penetration shape factor, weld reinforcement form factor, area of reinforcement, area of penetration and percentage dilution	L_{18}	Grey Relational Analysis
Al-Refaie, *et al*. (2008)	To optimize stainless steel 304 (SUS 304) wire drawing process.	(SUS 304) wire drawing	Die life and wire tensile strength	L_9	Grey Relational Analysis
Lin and Lee (2009)	To develop a novel process of EDM and to optimize the machining parameters	EDM	Material removal rate, electrode wear rate, and surface roughness	L_{18}	Grey Relational Analysis
Muruganantham, *et al*. (2009)	To determine the optimal settings of the parameters that will maximize the performance of the machine vision system.	Machine vision system	Accuracy and repeatability of calibration and DIPE	L_{27}	Desirability Function Approach
Chen, *et al*. (2010)	To examine the optimization of the process parameters of GZO films	GZO films	Deposition rate, electrical resistively, optical transmittance	L_{18}	Grey Relational Analysis
Nandi, *et al*. (2010)	To search an optimal process environment, capable of producing desired high quality submerged arc weldment.	Submerged arc weldment	Depth of penetration, reinforcement, bead width and percentage dilution	L_{25}	Grey Relational Analysis and Desirability Function Approach
Antony (2000)	To determine the optimal settings of factors (or process variables) this will simultaneously optimize several responses.	submerged arc-welding	Deposition rate and dilution	L_8	PCA
Tong (2005)	To obtain the optimal factor/level combination for multiple responses.	Chemical-mechanical polishing of copper	Removal rate, non-uniformity, and TaN/Cu selectivity	L_{18}	PCA

Table 2.2: Continued

Paper	Objective	Process	Quality Characteristics	OA	Optimization Approach
Fung and Kang (2005)	To optimize the injection-molding process for friction properties of fiber-reinforced polybutylene terephthalate.	Injection-molding	Friction coefficient and surface roughness	L_9	PCA
Huang and Lin (2008)	To develop a method for setting the robust injection molding parameters of multi-response.	Injection molding	Maximal shear stress, maximal warpage, and maximal volumetric shrinkage rate.	L_{18}	PCA
Gauri and Chakraborty (2009)	To optimize the WEDM process.	WEDM	Cutting speed, surface roughness, and dimensional deviation	L_{18}	PCA
Tarng, *et al*. (2000)	To optimize the submerged arc welding process with multi-response.	Submerged arc welding	Deposition rate and the dilution	L_8	Fuzzy-Based Method
Palanikumar (2006)	To optimize of machining parameters in turning glass fiber reinforced polymer.	Glass fiber reinforced polymer	Metal removal rate, tool wear, and surface roughness	L_{27}	Fuzzy-Based Method
Wang and Jean (2006)	To optimize the cobalt-based hardfacing in carbon steel.	Carbon steel	Vicker hardness and dilution percentage	L_{18}	Fuzzy-Based Method
Fang, *et al*. (2008)	To optimize non-image optics; a miniature optical engine.	Non-image optics	Luminous flux, illumination uniformity performance, and packing size	L_{18}	PCA with Fuzzy Logic

Table 2.2: Continued

Paper	Objective	Process	Quality Characteristics	OA	Optimization Approach
Lin and Lin (2005)	To optimize the electrical discharge machining process with multi-response.	EDM	Electrode wear ratio, material removal rate, and surface roughness	L_9	Grey-Fuzzy logic
(Chang and Lu, 2007)	To optimize cutting-parameter design of heavy cutting in side milling for SUS 304 stainless steel.	SUS 304 stainless steel	Tool life and metal removal rate	L_9	Grey-Fuzzy Logic
Liu, *et al*. (2009)	To achieve the optimization of multi-response characteristics during the manufacturing process.	Light guide plate printing	Illumination, homogeny, value of variance, and ink thickness	L_8	Grey-Fuzzy Logic
Gopalsamy, *et al*. (2009)	To find optimum process parameters for end milling while hard machining of hardened steel.	End milling	Surface finish and tool life	L_{18}	Regression Method
Pal and Gauri (2010)	To examine the effectiveness of the various performance metrics for multi-response optimization.	Polysilicon deposition process	Surface defects, thickness, and deposition rate	L_{18}	Regression Method

Chapter Three

Research Methodology

This chapter proposes an efficient approach for optimizing the multi-response problem in the manufacturing applications on the Taguchi method utilizing fuzzy regression-based approach.

3.1. The Taguchi method and fuzzy regression in the manufacturing applications

As mentioned earlier, the Taguchi utilizes OA to obtain dependable information about the design parameter with minimum time and resources, and adopts S/N ratio to interpret experimental data and optimize performance. In many manufacturing processes, it is difficult to find probability distributions for dependent variables. However, if the behavior of processes is vague and the observed data is irregular, the statistical regression models have an unnaturally wide possibility range. A fuzzy regression approach in modeling manufacturing processes, which have a high degree of fuzziness, has the distinct advantage of being able to generate models using only a small number of experimental data sets.

In a fuzzy regression model, the deviations between the observed values and the estimated values are expressed as the possibilities that the system has. The relationship between dependent and independent variables is defined by using fuzzy concept rather than statistical concept. In other words, a fuzzy regression model aims to build a model which contains all observed data in the estimated fuzzy numbers. In contrast to the statistical regression, fuzzy regression is based on possibility theory rather than probability theory. It has been noted that fuzzy regression could be more effective than

statistical regression when the degree of fuzziness of systems is high (Tanaka *et al*. 1982).

In fact, many manufacturing processes tend to be very complex in behavior and have inherent system fuzziness such as fluctuation of process pressure and temperature due to environmental effects. Sometimes, the observed values from the processes may not be regular. Fuzzy regression has been found to be more appealing than statistical regression in estimating the relationship between the dependent variable and independent variables when a high degree of fuzziness is involved and only a few data sets are available for model building.

The proposed approach in the manufacturing application on the Taguchi method is intending to use optimally the available data and gives the decision maker the opportunity to intervene and to use his experience in order to improve the quality of predictions utilizing various directions for the extension of fuzzy regression analysis.

In this approach, two well-known membership functions; triangular and trapezoidal membership functions will be used to minimize the fuzziness of the fuzzy model that includes all the given data.

3.2. Desirability function and pay-off matrix

The desirability function approach (Derringer and Suich, 1980) is one of the most widely used methods in industry for the optimization of multi-response processes such as (Muruganantham *et al*., 2009), and (Nandi *et al*., 2010). It is based on the idea that the "quality" of a product or process that has multi-responses, with one of them outside of some "desired" limits, is completely unacceptable. The method finds operating conditions ***X*** that provide the "most desirable" response values.

For each response, $\eta_j(\boldsymbol{X})$, a desirability function, $d_j(\eta_j)$, assigns numbers between zero and one to the possible values of η_j, with $d_j(\eta_j)$ equals zero representing a completely undesirable value of η_j and $d_j(\eta_j)$ equals one representing a completely desirable or ideal response value. Depending on whether a particular response η_j is to be the STB, the NTB, and the LTB type responses, different desirability functions $d_j(\eta_j)$ can be used.

Then a pay-off matrix will be used in the proposed approach to get all possible combinations of alternative responses, where a pay-off matrix is a decision analysis tool that summarizes pros and cons of a decision in a tabular form. It lists payoffs (negative or positive returns) associated with all possible combinations of alternative actions (under the decision maker's control) and external conditions (not under decision maker's control) (Leonard and Kazmier, 2009).

Consequently, the proposed approach for optimizing the multi-response in the manufacturing application on the Taguchi method utilizing fuzzy regression-based approach is outlined in the following steps:

***Step* 1**: Suppose q responses are of main concern measured in each experiment. To investigate v factors concurrently, assuming n experiments are conducted utilizing Taguchi's OA. In order to include all fuzziness in the observations values for each response, calculate the S/N ratio, η_{ijr}, at experiment i (i=1,…, n) for replicate r of response j (j=1,..,q), using an appropriate equation from the following formulas:

$$\eta_{ijr} = \begin{cases} -10\log_{10}(y_{ijr}^2) & \text{for STB,} \\ 10\log_{10}(s_{ijr}^2 / \bar{y}_{ijr}^2) & \text{for NTB,} \qquad \forall i,\ \forall j,\ \forall r \\ -10\log_{10}(1/y_{ijr}^2) & \text{for LTB,} \end{cases} \tag{3.1}$$

where $\bar{y}_i$ and s_i are the estimated average and standard deviation of y_{ir} replicates at the *i*th experiment.

***Step* 2**: Determine the optimal factor setting for *r*th replicate of response *j* using S/N ratio. Let $\bar{\eta}_{lfr}$ denotes the average of η_i values at level *l* of factor *f* for the *r*th replicate. Calculate the $\bar{\eta}_{lfr}$ values for all factor levels. For a factor *f*, decide the optimal level as the level that maximizes the $\bar{\eta}_{lfr}$ for this factor.

***Step* 3**: Obtain the multiple linear regressions η_{jr} for all factor level combinations using the calculated η_{ijr} in step 1. Let η_{jr}, denotes the S/N ratio for *r*th replicate of response *j*. That is,

$$\eta_{jr} = \beta_{0r} + \sum_{f=1}^{v} \beta_{fr} X_f + \sum_{f=1}^{v} \beta_{ffr} X_f^2 + \sum_{g<f} \sum \beta_{fgr} X_f X_g + \varepsilon, \quad r = 1, 2, ..., k \qquad (3.2)$$

where, X_f, X_f^2, and $X_f X_g$ the independent factor variables, and the coefficients, $\beta_f, \beta_{fg}, \beta_{ff}$, are crisp values, and ε is random error observed in the response value.

***Step* 4**: Determine the best-fit models for describing the functional relationship between the S/N ratio for response *j* and process factors.

a. If the triangular membership function is used to cover all data. Then, the triangular fuzzy regression $\tilde{\eta}_j$ based on mean and variance of coefficient of the obtained regression will be:

$$\tilde{\eta}_j = \tilde{\beta}_0 + \sum_{f=1}^{v} \tilde{\beta}_f X_f + \sum_{f=1}^{v} \tilde{\beta}_{ff} X_f^2 + \sum_{g<f} \sum \tilde{\beta}_{fg} X_f X_g + \varepsilon \qquad \forall j, \forall f \qquad (3.3)$$

Let $\tilde{\beta} = (\beta^l, \beta^m, \beta^u)$ are triangular fuzzy coefficients. Obtain β^l, β^m, and β^u as follows:

$$\tilde{\beta} = \begin{cases} \beta^{\mathrm{m}} = \mathrm{Average}(\beta_1, ..., \beta_{\mathrm{k}}) \\ \beta^{\mathrm{l}} = \beta^{\mathrm{m}} - \mathrm{s} \\ \beta^{\mathrm{u}} = \beta^{\mathrm{m}} + \mathrm{s} \end{cases} \tag{3.4}$$

where s is standard deviation of $(\beta_1, ..., \beta_k)$.

b. If the trapezoidal membership function is used to cover all data. Then, the trapezoidal fuzzy regression $\tilde{\eta}_j$ based on mean and variance of coefficient of the obtained regression will be as in Eq. (3.3). Let $\tilde{\beta} = (\beta^l, \beta^b, \beta^c, \beta^u)$ are trapezoidal fuzzy coefficients. Obtain $\beta^l, \beta^b, \beta^c$ and β^u as follows:

$$\tilde{\beta} = \begin{cases} \beta^{\mathrm{m}} = \mathrm{Average}(\beta_1, ..., \beta_{\mathrm{k}}) \\ \beta^{\mathrm{l}} = \beta^{\mathrm{m}} - \mathrm{s} \\ \beta^{\mathrm{b}} = \beta^{\mathrm{m}} - \lambda \mathrm{s} \\ \beta^{\mathrm{c}} = \beta^{\mathrm{m}} + \lambda \mathrm{s} \\ \beta^{\mathrm{u}} = \beta^{\mathrm{m}} + \mathrm{s} \end{cases} \tag{3.5}$$

where s is standard deviation of $(\beta_1, ..., \beta_k)$, λ is a positive constant $0 \le \lambda \le 1$, chosen by an expert depending on experience about the repetitiveness of the proposed data. For instance a large λ means that the expert has a poor opinion about their repetitiveness.

Based on the results of step 2, the optimal triangular /trapezoidal fuzzy factor levels $\tilde{X}^*_{fj}$ for a factor f and for response *j* constituted based on mean and variance of the combination of optimal factor levels.

***Step* 5**: Obtain all possible combinations of alternative responses, the values of pay-off matrix for responses, $\tilde{\eta}_j(\tilde{X}^*_q)$, is constituted in Table 3.1. Let $\tilde{X}^*_q$ be the optimal fuzzy factor levels of the *q*th response and $\tilde{\eta}_j(\tilde{X}^*_q)$ be *j*th response value by replacing the optimal fuzzy factor levels of *q*th response.

Table 3.1: Pay-off matrix values for responses.

	$\tilde{\eta}_1(X)$	…	$\tilde{\eta}_j(X)$	…	$\tilde{\eta}_q(X)$
$\tilde{X}_1^*$	$\tilde{\eta}_1(\tilde{X}_1^*)$	…	$\tilde{\eta}_j(\tilde{X}_1^*)$	…	$\tilde{\eta}_q(\tilde{X}_1^*)$
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
$\tilde{X}_q^*$	$\tilde{\eta}_1(\tilde{X}_q^*)$	…	$\tilde{\eta}_j(\tilde{X}_q^*)$	…	$\tilde{\eta}_q(\tilde{X}_q^*)$

***Step* 6**: Provide the "most desirable" response values, the desirability function for LTB type response will be calculated for all response type to reduce computation procedure using Eq. (3.6).

$$\tilde{d}_j(\tilde{\eta}_j(X)) = \begin{cases} 0, & \tilde{\eta}_j \le \eta_{\min} \\ \dfrac{\tilde{\eta}_j(X) - \eta_{\min}}{\eta_{\max} - \eta_{\min}}, & \eta_{\min} \le \eta_j(X) \le \eta_{\max} \\ 1, & \tilde{\eta}_j \ge \eta_{\max} \end{cases} \tag{3.6}$$

Then, the pay-off matrix for desirability values is constituted in Table 3.2.

Table 3.2: Pay-off matrix for desirability values.

	$\tilde{d}_1(\tilde{\eta}_1(X))$	…	$\tilde{d}_j(\tilde{\eta}_j(X))$	…	$\tilde{d}_q(\tilde{\eta}_q(X))$
$\tilde{X}_1^*$	$\tilde{d}_1(\tilde{\eta}_1(\tilde{X}_1^*))$	…	$\tilde{d}_j(\tilde{\eta}_j(\tilde{X}_1^*))$	…	$\tilde{d}_q(\tilde{\eta}_q(\tilde{X}_1^*))$
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
$\tilde{X}_q^*$	$\tilde{d}_1(\tilde{\eta}_1(\tilde{X}_q^*))$	…	$\tilde{d}_j(\tilde{\eta}_j(\tilde{X}_q^*))$	…	$\tilde{d}_q(\tilde{\eta}_q(\tilde{X}_q^*))$

From Table 3.2, the following values as the upper and lower limit for the desirability functions are determined:

$$\tilde{U}_j = \tilde{d}_j(\tilde{\eta}_j(\tilde{X}_j^*)), \quad j=1,..,q$$

$$\tilde{L}_j = Min\left\{\tilde{d}_j(\tilde{\eta}_j(\tilde{X}_1^*)),...,\tilde{d}_j(\tilde{\eta}_j(\tilde{X}_q^*))\right\}, \quad j=1,..,q$$

***Step* 7**: Define the deviation function for each response, so it is desirable to decrease it. So let $D_j(X)$, denotes the deviation function to be minimized, and then calculate the $D_j(X)$ based on the chosen membership function from the following formulas:

$$D_j(X) = \begin{cases} (\eta_j^u(X) - \eta_j^m(X)), & \text{For triangular membership function} \\ \dfrac{\eta_j^u(x) - \eta_j^c(x)}{(1-\lambda)}, & \text{For trapezoidal membership function} \end{cases}, j = 1,\ldots,q. \qquad (3.7)$$

Then, the pay-off matrix for deviation values is constituted in Table 3.3.

Table 3.3: Pay-off matrix for deviation values.

	$\tilde{D}_1(X)$	$\cdots$	$\tilde{D}_j(X)$	$\cdots$	$\tilde{D}_q(X)$
$\tilde{X}_1^*$	$\tilde{D}_1(\tilde{X}_1^*)$	$\cdots$	$\tilde{D}_j(\tilde{X}_1^*)$	$\cdots$	$\tilde{D}_q(\tilde{X}_1^*)$
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
$\tilde{X}_q^*$	$\tilde{D}_1(\tilde{X}_q^*)$	$\cdots$	$\tilde{D}_j(\tilde{X}_q^*)$	$\cdots$	$\tilde{D}_q(\tilde{X}_q^*)$

Thus, from Table 3.3 the followings are expressed as:

$$\tilde{P}_j = \tilde{D}_j(\tilde{X}_j^*), \tilde{Q}_j = Max\left\{\tilde{D}_j(\tilde{X}_1^*),\ldots,\tilde{D}_j(\tilde{X}_q^*)\right\}, \quad j=1,..,q$$

***Step* 8**: Formulate the final model, which is a two-objective model as shown in Eq. (3.8):

$$\begin{aligned} & Max\left\{\tilde{d}_1(X),\ldots,\tilde{d}_j(X)\right\}, \qquad j = 1,\ldots,q \\ & Min\left\{D_1(X),\ldots,D_j(X)\right\}, \qquad j = 1,\ldots,q \\ & s.t \\ & X \in \text{[Factor Levels]} \end{aligned} \qquad (3.8)$$

Then, Eq. (3.8) will be converted to single objective, by introducing two functions $\tilde{S}_j(X)$ and $\tilde{T}_j(X)$.

a. For the triangular membership function, let:

$\tilde{S}_j(X) = (S_j^l(X), S_j^m(X), S_j^u(X))$

and

$\tilde{T}_j(X) = (T_j^l(X), T_j^m(X), T_j^u(X))$

b. For the trapezoidal membership function, let:

$\tilde{S}_j(X) = (S_j^l(X), S_j^b(X), S_j^c(X), S_j^u(X))$

and

$\tilde{T}_j(X) = (T_j^l(X), T_j^b(X), T_j^c(X), T_j^u(X))$

Where $\tilde{S}_j(X)$ and $\tilde{T}_j(X)$, indicating the degrees of satisfaction from desirability and robustness, respectively. The introducing two functions, Eqs. (3.9) and (3.10), for the *j*th response are:

$$\tilde{S}_j(X) = \begin{cases} 0, & \tilde{d}_j(X) \le \tilde{L}_j \\ \dfrac{\tilde{d}_j(X) - \tilde{L}_j}{\tilde{U}_j - \tilde{L}_j}, & \tilde{L}_j \le \tilde{d}_j \le \tilde{U}_j \\ 1, & \tilde{d}_j(X) \ge \tilde{U}_j \end{cases} \tag{3.9}$$

and

$$\tilde{T}_j(X) = \begin{cases} 1, & \tilde{D}_j(X) \le \tilde{P}_j \\ \dfrac{\tilde{Q}_j - \tilde{D}_j(X)}{\tilde{Q}_j - \tilde{P}_j}, & \tilde{P}_j \le \tilde{D}_j(X) \le \tilde{Q}_j \\ 0, & \tilde{D}_j(X) \ge \tilde{Q}_j \end{cases} \tag{3.10}$$

Then, the objective will be to maximize these two functions as in Eq. (3.11), to obtain $\tilde{S}_j(X)$ and $\tilde{T}_j(X)$. That is,

$$\begin{aligned} &Max \quad \tilde{S}_j(X) \qquad j=1,\dots,q \\ &Max \quad \tilde{T}_j(X) \qquad j=1,\dots,q \\ &s.t \\ &X \in [\text{Factor Levels}] \end{aligned} \tag{3.11}$$

To convert two-objective model to single objective Zimmerman Max-Min operator will be applied, which maximizes the minimum degree of satisfaction from two objectives (Zimmermann, 1987).

For triangular/trapezoidal membership function, let

$$Min \quad \tilde{S}_j(X) = \tilde{S}$$

and

$$Min \quad \tilde{T}_j(X) = \tilde{T}$$

Then, the problem is formulated in Eq. (3.12) as:

$$\begin{aligned} &Max \quad \tilde{S}(X) \\ &Max \quad \tilde{T}(X) \\ &s.t \\ &X \in [\text{Factor Levels}] \end{aligned} \tag{3.12}$$

Subject to:

$$\tilde{S} \le \frac{\tilde{d}_j(X) - \tilde{L}_j}{\tilde{U}_j - \tilde{L}_j}, \text{then } \tilde{d}_j(X) - \tilde{S}(\tilde{U}_j - \tilde{L}_j) \ge \tilde{L}_j \qquad , j=1,\dots,q$$

$$\tilde{T} \le \frac{\tilde{Q}_j - \tilde{D}_j(X)}{\tilde{Q}_j - \tilde{P}_j}, \text{then } \tilde{D}_j(X) + \tilde{T}(\tilde{Q}_j - \tilde{P}_j) \le \tilde{Q}_j \qquad , j=1,\dots,q$$

Finally, let w_1 and w_2 indicate the important weights for desirability and robustness expressed by user based on cost, quality loss, and warranty. The final model with only one objective will be expressed as in Eq. (3.13) as:

$$
\begin{aligned}
&Max \quad w_1\tilde{S} + w_2\tilde{T} \\
&s.t \\
&\tilde{d}_j(X) - \tilde{S}(\tilde{U}_j - \tilde{L}_j) \ge \tilde{L}_j \qquad , j = 1,\dots,q \\
&\tilde{D}_j(x) + \tilde{T}(\tilde{Q}_j - \tilde{P}_j) \le \tilde{Q}_j \qquad , j = 1,\dots,q \\
&w_1 + w_2 = 1 \\
&0 \le \tilde{S} \le 1 \\
&0 \le \tilde{T} \le 1 \\
&X \in [\text{Factor Levels}].
\end{aligned} \tag{3.13}
$$

The final model is categorized to:

a. Three types for triangular membership function: model l, model m, and model u.

b. Four types for trapezoidal membership function: model l, model b, model c, and model u.

***Step* 9**: Solve the models by computer software. That is,

a. For the triangular membership function, the optimal factor levels of models l, m, and u, are determined. Then, substituting the values of factor levels obtained from three models in each response equations, the optimal triangular fuzzy responses can be obtained.

b. For the trapezoidal membership function, the optimal factor levels of models l, b, c and u, are determined. Then, substituting the values of factor levels obtained from four models in each response equations, the optimal trapezoidal fuzzy responses can be obtained.

Chapter Four

Case Studies

In this chapter, two case studies, including Inconel on machining of CNC of wire electro-discharge machining (WEDM) process, and the sputtering process parameters of GZO films, which were investigated in previous literature, will be provided for illustrating the proposed approach. A validation experiment is also carried out to demonstrate the effectiveness of the proposed approach.

4.1. Optimization of Inconel on machining of CNC WEDM process

Ramakrishnan *et al.* (2008) modeled the multiple response optimization of Inconel 718 on machining of CNC WEDM process. Four process factors were investigated concurrently using the L_9 orthogonal array, including: (X_1) pulse in time, (X_2) delay time, (X_3) wire feed speed, and (X_4) ignition current, on two responses: material removal rate (MRR) and surface roughness (SR) of WEDM process, where (y_1, LTB) and (y_2, STB), respectively, the experimental data are displayed in Table 4.1, and factors and their levels for experiment can be seen in Table 4.2.

Table 4.1: Orthogonal array and the experimental data for WEDM process.

Exp. *i*	**Factor Levels**				**MRR***		**SR***	
	X_1	X_2	X_3	X_4	y_{11}	y_{12}	y_{21}	y_{22}
1	1	1	1	1	46	46	3.2	3.1
2	1	2	2	2	48	47	3.3	3.2
3	1	3	3	3	42	41	3.3	3.3
4	2	1	2	3	56	55	3.8	3.7
5	2	2	3	1	50	49	3.4	3.5
6	2	3	1	2	52	53	3.2	3.3
7	3	1	3	2	70	71	4.2	4
8	3	2	1	3	74	73	3.8	3.5
9	3	3	2	1	64	64	3.4	3.3

* The second replicate in this case is generated for illustration.

Table 4.2: Factors and their levels for WEDM experiment.

Factor	Process variables	Level 1	Level 2	Level 3
X_1	Pulse on time (μs)	0.6	0.8	1.2
X_2	Wire feed speed (m/min)	4	6	8
X_3	Ignition current (A)	8	12	15
X_4	Mean MRR (mm^2/min)	8	12	16

The proposed approach is employed to optimize the two responses as follows:

***Step* 1**: Let η_{i1r} and η_{i2r}, denote the S/N ratio for MRR and SR at experiment i (i =1,…, 9) with replicate r (r =1, 2), respectively. Then, the η_{i1r} and η_{i2r} are calculated respectively for each replicate as follows:

$$\eta_{i1r} = -10\log_{10}(1/y_{i1r}^2) \qquad \text{for MRR} \quad i = 1,...,9 \quad r = 1,2 \tag{4.1}$$

$$\eta_{i2r} = -10\log_{10}(y_{i2r}^2) \qquad \text{for SR} \quad i = 1,...,9 \quad r = 1,2 \tag{4.2}$$

The obtained η_{i1r} and η_{i2r} for each replicate are summarized in Table 4.3. For illustration, at the first experiment (i=1), the η_{111}, and η_{121} are respectively calculated as:

$$\eta_{111} = -10\log_{10}\left(\frac{1}{(46)^2}\right) = 33.26$$

$$\eta_{121} = -10\log_{10}(3.2)^2 = -10.103$$

The values of η_{i1r}, η_{i2r} for the other 8 experiments are calculated similarly.

***Step* 2**: For MRR, the $\bar{\eta}_{lfr}$ values for the first (r=1) and second (r=2) replicates are calculated for all factor levels and displayed in Table 4.4. The combination of optimal factor levels for the first replicate that maximizes the $\bar{\eta}_{lf1}$ is identified as $X_{1(3)}X_{2(1)}X_{3(1)}X_{4(2)}$, which are the same for the second replicate. Similarly, for SR the $\bar{\eta}_{lfr}$ values for the two replicates are calculated for all factor levels and displayed in

Table 4.5, where the combination of optimal factor levels, is identified as $X_{1(1)} X_{2(3)} X_{3(1)} X_{4(1)}$, which is found the same for each replicate of SR.

Table 4.3: The experimental results for each replicates for WEDM.

Exp. i	η_{i11}	η_{i12}	η_{i21}	η_{i22}
1	33.260	33.260	-10.103	-9.827
2	33.620	33.442	-10.370	-10.103
3	32.460	32.260	-10.370	-10.370
4	34.960	34.810	-11.596	-11.364
5	33.980	33.800	-10.630	-10.881
6	34.320	34.490	-10.103	-10.370
7	36.900	37.030	-12.465	-12.041
8	37.380	37.270	-11.596	-10.881
9	36.120	36.120	-10.630	-10.370

Table 4.4: The optimal factor levels for the two replicates of MRR.

Factor (f)	**Replicate** r=1			**Replicate** r=2		
	Level 1	Level 2	Level 3	Level 1	Level 2	Level 3
X_1	33.115	34.421	36.803	32.984	34.366	36.805
X_2	35.040	34.996	34.303	35.029	34.837	34.288
X_3	34.987	34.904	34.449	35.002	34.791	34.362
X_4	34.453	34.949	34.938	34.394	34.984	34.777

Table 4.5: The optimal factor levels for the two replicates of SR.

Factor (f)	**Replicate** r=1			**Replicate** r=2		
	Level 1	Level 2	Level 1	Level 2	Level 1	Level 2
X_1	-10.281	-10.776	-11.563	-10.100	-10.872	-11.098
X_2	-11.388	-10.865	-10.368	-11.078	-10.622	-10.370
X_3	-10.600	-10.865	-11.155	-10.360	-10.612	-11.098
X_4	-10.454	-10.979	-11.187	-10.360	-10.838	-10.872

***Step* 3**: The multiple linear regression equations for η_{11} and η_{12} are respectively written as:

$$\eta_{11} = 30.7153 + 6.12x_1 - 0.18435x_2 - 0.0738x_3 + 0.06063x_4 \qquad (4.3.a)$$

R-Sq (adj) = 96.1%

and

$$\eta_{12} = 30.8143 + 6.3296x_1 - 0.18523x_2 - 0.08945x_3 + 0.04778x_4 \qquad (4.3.b)$$

R-Sq (adj) = 96.3%

Whereas, the multiple linear regression equations η_{21} and η_{22} are represented respectively by:

$$\eta_{21} = -8.5573 - 2.1129x_1 + 0.25507x_2 - 0.07849x_3 - 0.09164x_4 \qquad (4.4.a)$$

R-Sq (adj) = 97.6%

and

$$\eta_{22} = -8.4742 - 1.5055x_1 + 0.1768x_2 - 0.10314x_3 - 0.06403x_4 \qquad (4.4.b)$$

R-Sq (adj) = 70.7%

Case I: Steps 4-9, are conducted for triangular membership function, and are described as follow:

***Step* 4**: The triangular fuzzy regression $\tilde{\eta}_1$ and $\tilde{\eta}_2$ are obtained for the two responses and expressed as:

$\tilde{\eta}_1 =$ (30.6948, 30.7648, 30.8348) + (6.0766, 6.2248, 6.3730) X_1 + (-0.1854, -0.18479, -0.18417) X_2 + (-0.0927, -0.08163, -0.0706) X_3 + (0.04512, 0.0542, 0.0633) X_4. (4.5)

and

$\tilde{\eta}_2 =$ (-8.5745, -8.51575, -8.4570) + (-2.2387, -1.8093, -1.3797) X_1 + (0.1606, 0.21595, 0.2713) X_2 + (-0.1083, -0.0908, -0.0734) X_3 + (-0.0974, -0.07785, -0.0583) X_4. (4.6)

For illustration for the first response using Eqs. (4.3.a) and (4.3.b), respectively, to calculate $\tilde{\beta}_0$ *(l, m, u)* utilizing Eq. (3.4) as follows:

β_0^m = Average (30.7153, 30.8143) = 30.7648.

Let *s*, is the calculated standard deviation:

s = 0.070004.

Then, the β^l and β^u are respectively calculated as follows:

$\beta^l = \beta_0^m - s$ = 30.6948

$\beta^u = \beta_0^m + s$ = 30.8348

The remaining triangular fuzzy coefficients are calculated similarly for both responses. Based on the results of step 2, the optimal fuzzy factor levels $\tilde{X}_{fj}^*$ that optimize MMR response are as follows:

$\tilde{X}_{11}^*$ = (1.2, 1.2, 1.2), $\tilde{X}_{21}^*$ = (4, 4, 4), $\tilde{X}_{31}^*$ = (8, 8, 8), $\tilde{X}_{41}^*$ = (16, 16, 16).

Whereas the optimal fuzzy factor levels $\tilde{X}_{fj}^*$ that optimize SR response are as follows:

$\tilde{X}_{12}^*$ = (0.6, 0.6, 0.6), $\tilde{X}_{22}^*$ = (8, 8, 8), $\tilde{X}_{32}^*$ = (8, 8, 8), $\tilde{X}_{42}^*$ = (8, 8, 8).

For illustration, the optimal level founded for MRR for the first factor, $\tilde{X}_{11}^*$, is (1.2, 1.2, 1.2) on level three, $X1_{(3)}$, which is the same for the second replicate, so the standard deviation equals zero, which is the same for the other factors.

By substituting the optimal factor levels for the two responses in Eqs. (4.5) and (4.6), the values of the optimal triangular fuzzy responses are respectively expressed as:

$\tilde{\eta}_1^*$ = (37.2267, 37.7095, 38.1936)

and

$\tilde{\eta}_2^*$ = (-10.2785, -9.2223, -8.1680)

***Step* 5**: The values of pay-off matrix $\tilde{\eta}_j(X)$ for both responses are shown in Table 4.6.

Table 4.6: Pay-off matrix values for MRR and SR.

	$\tilde{\eta}_1(X)$	$\tilde{\eta}_2(X)$
$\tilde{X}_1^*$	(37.2267, 37.7095, 38.1936)	(-13.0433, -11.7951, -10.5474)
$\tilde{X}_2^*$	(32.4776, 32.8020, 33.1266)	(-10.2785, -9.2229, -8.1680)

For illustration, the second cell corresponding to the first response $\tilde{\eta}_1(\tilde{X}_2^*)$, which is equal (32.4776, 32.8020, 33.1266) is calculated by substituting the optimal factor levels for the second response in Eq. (4.5):

$\tilde{\eta}_1 =$ (30.6948, 30.7648, 30.8348) + (6.0766, 6.2248, 6.3730) × (0.6, 0.6, 0.6) + (-0.1854, - 0.18479, -0.1842) × (8, 8, 8) + (-0.0927, -0.08163, -0.0706) × (8, 8, 8) + (0.0452, 0.0542, 0.0633) × (8, 8, 8) = (32.4776, 32.8020, 33.1266)

The $\tilde{\eta}_2(\tilde{X}_1^*)$ is calculated by substituting the optimal factor levels for the second response in Eq. (4.6)

***Step* 6**: The desirability function for each response is calculated using Eq. (3.6), where $\eta_{\min} = (30, 30, 30)$ and $\eta_{\max} = (40, 40, 40)$ are determine as an acceptable ranges for MRR response, while for SR response the acceptable ranges are $\eta_{\min} = (-13, -13, -13)$ and $\eta_{\max} = (-7, -7, -7)$. Then, the pay-off matrix for desirability values is shown in Table 4.7.

Table 4.7: Pay-off matrix for desirability values for MRR and SR.

	$\tilde{d}_1(\tilde{\eta}_1(X))$	$\tilde{d}_2(\tilde{\eta}_2(X))$
$\tilde{X}_1^*$	(0.7227, 0.7710, 0.8194)	(0, 0.2008, 0.4088)
$\tilde{X}_2^*$	(0.2478, 0.2802, 0.3127)	(0.4536, 0.6295, 0.8053)

For illustration, the second cell $\tilde{d}_1(\tilde{\eta}_1(\tilde{X}^*_2))$ corresponding to the first response is calculated by substituting the value $\tilde{\eta}_1(\tilde{X}^*_2)$ in Eq. (3.6) as follows:

$$\tilde{d}_1(\tilde{\eta}_1(\tilde{X}^*_2)) = \begin{cases} 0, & \tilde{\eta}_1(X) \le (30, 30, 30) \\ \dfrac{(32.4776, 32.8020, 33.1266) - (30, 30, 30)}{(40, 40, 40) - (30, 30, 30)}, & (30, 30, 30) \le \eta_1(X) \le (40, 40, 40) \\ 1, & \tilde{\eta}_1(X) \ge (40, 40, 40) \end{cases}$$

The following values from Table 4.7 are determined as:

$$\tilde{U}_1 = \tilde{d}_{11} = (0.7227, 0.7710, 0.8194),\ \tilde{L}_1 = \min\{\tilde{d}_{11}, \tilde{d}_{21}\} = (0.2478, 0.2802, 0.3127)$$

$$\tilde{U}_2 = \tilde{d}_{22} = (0.4536, 0.6295, 0.8053),\ \tilde{L}_2 = \min\{\tilde{d}_{12}, \tilde{d}_{22}\} = (0, 0.2008, 0.4088)$$

***Step* 7**: The deviation function, $D_j(X)$, for each response is defined as:

$$D_j(X) = \eta_j^u - \eta_j^m(X) \qquad , j = 1, 2.$$

The fuzzy deviation functions for MRR and SR are then expressed respectively as:

$$\tilde{D}_1(X) = 0.07 + 0.1482\tilde{X}_1 + 0.0006\tilde{X}_2 + 0.011\tilde{X}_3 + 0.0092\tilde{X}_4. \qquad (4.7)$$

and

$$\tilde{D}_2(X) = 0.0588 + 0.4296\ \tilde{X}_1 + 0.0554\ \tilde{X}_2 + 0.0174\ \tilde{X}_3 + 0.0196\tilde{X}_4. \qquad (4.8)$$

The pay-off matrix for deviation values is constituted as shown in Table 4.8.

Table 4.8: Pay-off matrix for deviation values for MRR and SR.

	$\tilde{D}_1(X)$	$\tilde{D}_2(X)$
$\tilde{X}^*_1$	(0.4854, 0.4854, 0.4854)	(1.2487, 1.2487, 1.2487)
$\tilde{X}^*_2$	(0.3253, 0.3253, 0.3253)	(1.0558, 1.0558, 1.0558)

For example, the second cell of the first response $\tilde{D}_1(\tilde{X}^*_2)$, which is equal (0.3245, 0.3245, 0.3245) is calculated by substituting the optimal factor levels for the second response in Eq. (4.7) as follows:

$\tilde{D}_1(\tilde{X}_2^*)$ = 0.07 + 0.1482 (0.6, 0.6, 0.6) + 0.0006 (4, 4, 4) + 0.011 (8, 8, 8) + 0.0092 (0.6, 0.6, 0.6) = (0.3253, 0.3253, 0.3253).

From Table 4.8, the following values are expressed as:

$$\tilde{P}_1 = \tilde{D}_{11} = (0.4854, 0.4854, 0.4854), \tilde{Q}_1 = \max\left\{\tilde{D}_{11}, \tilde{D}_{21}\right\} = (0.4854, 0.4854, 0.4854).$$

$$\tilde{P}_2 = \tilde{D}_{22} = (1.0558, 1.0558, 1.0558), \ \tilde{Q}_2 = \max\left\{\tilde{D}_{12}, \tilde{D}_{22}\right\} = (1.2487, 1.2487, 1.2487).$$

***Step* 8**: The final model is constructed as

$$\begin{aligned} & Max\left\{\tilde{d}_1(X), \tilde{d}_2(X)\right\} \\ & Min\left\{D_1(X), D_2(X)\right\} \\ & s.t \\ & X = \left\{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4\right\} \in \text{[Factor Levels]} \end{aligned} \qquad (4.9)$$

Based on results of steps 6 and 7, and using Eq. (3.9), and Eq. (3.10), the following constituted as:

$$\tilde{S}_1(X) = \begin{cases} 0, & \tilde{d}_1(X) \le (0.2478, 0.2802, 0.3127) \\ \dfrac{\tilde{d}_1(X) - (0.2478, 0.2802, 0.3127)}{(0.7227, 0.7710, 0.8194) - (0.2478, 0.2802, 0.3127}, & (0.2478, 0.2802, 0.3127) \le \tilde{d}_1 \le (0.7227, 0.7710, 0.8194) \\ 1, & \tilde{d}_1(X) \ge (0.7227, 0.7710, 0.8194) \end{cases}$$

$$\tilde{S}_2(X) = \begin{cases} 0, & \tilde{d}_2(X) \le (0, 0.2008, 0.4088) \\ \dfrac{\tilde{d}_2(X) - (0, 0.2008, 0.4088)}{(0.4536, 0.6295, 0.8052) - (0, 0.2008, 0.4088)}, & (0, 0.2008, 0.4088) \le \tilde{d}_2 \le (0.4536, 0.6295, 0.8052) \\ 1, & \tilde{d}_2(X) \ge (0.4536, 0.6295, 0.8052) \end{cases}$$

and

$$\tilde{T}_1(X) = \begin{cases} 1, & \tilde{D}_1(X) \le (0.4854, 0.4473, 0.4473) \\ \dfrac{(0.4854, 0.4854, 0.4854) - \tilde{D}_1(X)}{(0.4854, 0.4854, 0.4854) - (0.4854, 0.4854, 0.4854)}, & (0.4854, 0.4854, 0.4854) \le \tilde{D}_1 \le (0.4854, 0.4854, 0.4854) \\ 0, & \tilde{D}_1(X) \ge (0.4854, 0.4854, 0.4854) \end{cases}$$

$$\tilde{T}_2(X) = \begin{cases} 1, & \tilde{D}_2(X) \le (1.0558,1.0558,1.0558) \\ \dfrac{(1.2487,1.2487,1.2487) - \tilde{D}_2(X)}{(1.2487,1.1687,1.1687) - (1.0558,1.0558,1.0558)}, & (1.0558,1.0558,1.0558) \le \tilde{D}_2 \le (1.2487,1.2487,1.2487) \\ 0, & \tilde{D}_2(X) \ge (1.2487,1.2487,1.2487) \end{cases}$$

Since it is desirable to maximize these two functions, the objective function is formulated as follows:

$$\begin{aligned} & Max \ \left\{\tilde{S}_1(X), \tilde{S}_2(X)\right\} \\ & Max \ \left\{\tilde{T}_1(X), \tilde{T}_2(X)\right\} \\ & s.t \\ & X = \left\{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4\right\} \in [\text{Factor Levels}] \end{aligned} \qquad (4.10)$$

By applying Zimmerman Max-Min operator, the two objectives is converted to single objective, this maximizes the minimum degree of satisfaction from the two objectives. Let $Min \quad \tilde{S}_j(x) = \tilde{S} = (S^l, S^m, S^u)$, $Min \quad \tilde{T}_j(x) = \tilde{T} = (T^l, T^m, T^u)$. Then, the problem is formulated as follows:

$$\begin{aligned} & Max \quad \tilde{S}(X) \\ & Max \quad \tilde{T}(X) \\ & s.t \\ & \tilde{d}_j(X) - \tilde{S}(\tilde{U}_j - \tilde{L}_j) \ge \tilde{L}_j \qquad , j = 1, 2. \\ & \tilde{D}_j(X) + \tilde{T}(\tilde{Q}_j - \tilde{P}_j) \le \tilde{Q}_j \qquad , j = 1, 2. \\ & X = \left\{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4\right\} \in [\text{Factor Levels}] \end{aligned} \qquad (4.11)$$

Finally, let $w_1 = w_2 = 0.5$, the final model using Eq. (3.13) is categorized to three types: model l, model m, and model u as shown below:

Model l **:**

$$
\begin{aligned}
&Max \quad 0.5\times S^{l} + 0.5\times T^{l} \\
&s.t \\
&0.06948 + 0.60766x_1^{l} - 0.01854x_2^{l} - 0.00927x_3^{l} + 0.00452x_4^{l} - 0.4749\,S^{l} \geq 0.2478 \\
&0.73758 - 0.37312x_1^{l} + 0.02677x_2^{l} - 0.01805x_3^{l} - 0.01623x_4^{l} - 0.4536\,S^{l} \geq 0 \\
&0.07000 + 0.1482x_1^{l} + 0.0006x_2^{l} + 0.011x_3^{l} + 0.0092x_4^{l} + 0\,T^{l} \leq 0.4854 \\
&0.0588 + 0.4296x_1^{l} + 0.0554x_2^{l} + 0.0174x_3^{l} + 0.0196x_4^{l} + 0.1929\,T^{l} \leq 1.2487 \\
&w_1 + w_2 = 1 \\
&0 \leq S^{l}, T^{l} \leq 1 \\
&X = \left\{x_1^{l}, x_2^{l}, x_3^{l}, x_4^{l}\right\} \in \text{[Factor Levels]}
\end{aligned}
$$

Model m **:**

$$
\begin{aligned}
&Max \quad 0.5\times S^{m} + 0.5\times T^{m} \\
&s.t \\
&0.07648 + 0.62248x_1^{m} - 0.01848x_2^{m} - 0.008163x_3^{m} + 0.00542x_4^{m} - 0.4908\,S^{m} \geq 0.2802 \\
&0.74738 - 0.30155x_1^{m} + 0.03599x_2^{m} - 0.01513x_3^{m} - 0.01298x_4^{m} - 0.4287\,S^{m} \geq 0.2008 \\
&0.0700 + 0.1482x_1^{m} + 0.0006x_2^{m} + 0.011x_3^{m} + 0.0092x_4^{m} + 0\,T^{m} \leq 0.4854 \\
&0.0588 + 0.4296x_1^{m} + 0.0554x_2^{m} + 0.0174x_3^{m} + 0.0196x_4^{m} + 0.1929\,T^{m} \leq 1.2487 \\
&w_1 + w_2 = 1 \\
&0 \leq S^{m}, T^{m} \leq 1 \\
&X = \left\{x_1^{m}, x_2^{m}, x_3^{m}, x_4^{m}\right\} \in \text{[Factor Levels]}
\end{aligned}
$$

Model u **:**

$$
\begin{aligned}
&Max \quad 0.5\times S^{u} + 0.5\times T^{u} \\
&s.t \\
&0.08348 + 0.6373x_1^{u} - 0.01842x_2^{u} - 0.00706x_3^{u} + 0.00633x_4^{u} - 0.5067\,S^{u} \geq 0.3127 \\
&0.7572 - 0.22995x_1^{u} + 0.04522x_2^{m} - 0.01223x_3^{u} - 0.00972x_4^{u} - 0.3965\,S^{u} \geq 0.4088 \\
&0.0700 + 0.1482x_1^{u} + 0.0006x_2^{u} + 0.011x_3^{u} + 0.0092x_4^{u} + 0\,T^{u} \leq 0.4854 \\
&0.0588 + 0.4296x_1^{u} + 0.0554x_2^{u} + 0.0174x_3^{u} + 0.0196x_4^{u} + 0.1929\,T^{u} \leq 1.2487 \\
&w_1 + w_2 = 1 \\
&0 \leq S^{u}, T^{u} \leq 1 \\
&X = \left\{x_1^{u}, x_2^{u}, x_3^{u}, x_4^{u}\right\} \in \text{[Factor Levels]}
\end{aligned}
$$

***Step* 9**: The three models on different weights combination (0.1, 0.9), (0.2, 0.8), (0.3, 0.7), (0.4, 0.6), and (0.5, 0.5) for desirability and robustness could be solved, then the optimal factor levels of models l, m, and u are determined as shown in Table 4.9.

Table 4.9: The optimal factor levels for WEDM process.

	Model *l*	Model *m*	Model *u*
X_1^*	0.9330	0.9434	0.9624
X_2^*	5.1902	5.3398	5.4184
X_3^*	8	8	8
X_4^*	8	8	8

Substituting the values the of factor levels from model *l*, model *m*, and model *u*, respectively in Eq. (4.4), and in Eq. (4.5), the results are respectively expressed as:

$\tilde{\eta}_1^*$ = (35.0220, 35.4311, 35.9117)

$\tilde{\eta}_2^*$ = (-11.4753, -10.4187, -9.3684)

By converting the $\tilde{\eta}_1^*$ to $\tilde{y}_1^*$ using Eq. (4.1) and converting $\tilde{\eta}_2^*$ to $\tilde{y}_2^*$ using Eq. (4.2), the optimal fuzzy responses are respectively obtained as:

$\tilde{y}_1^*$ = (56.3767, 59.0956, 62.4576)

$\tilde{y}_2^*$ = (2.9405, 3.3184, 3.7477)

The factor levels for initial condition were set at the combinations $X_{1(2)}X_{2(2)}X_{3(2)}X_{4(2)}$. By substituting the values of factors levels for this combination in Eqs. (4.4) and (4.5), and converting the results to $\tilde{y}_1$ and $\tilde{y}_2$ using Eqs. (4.1) and (4.2), respectively, the obtained triangular fuzzy responses at initial combinations of factor levels are written as:

$\tilde{y}_1$ = (49.3895, 51.9200, 54.5796)

$\tilde{y}_2$ = (2.9910, 3.4242, 3.9221)

Plotting the MRR triangular fuzzy response at initial combinations of factor levels, $\tilde{y}_1$, and at optimal combinations of factor levels, $\tilde{y}_1^*$, both are represented, respectively, in Figures 4.1 and 4.2. Similarly plotting the SR triangular fuzzy response, $\tilde{y}_2$, initial and $\tilde{y}_2^*$, optimal are represented, respectively, in Figures 4.3, and 4.4.

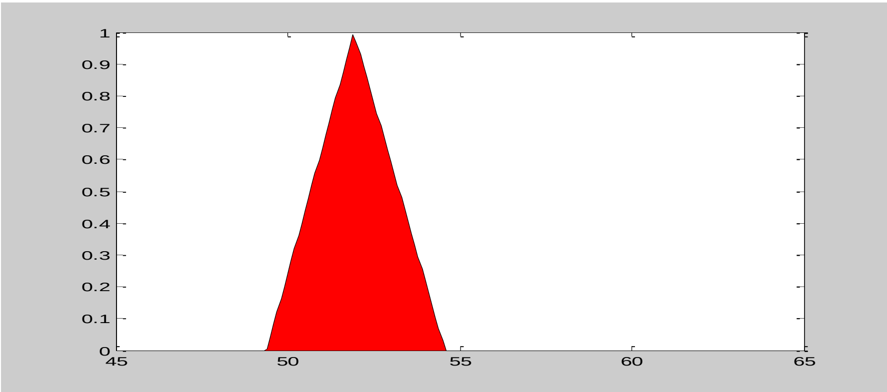


Figure 4.1: The MRR triangular fuzzy response at initial combination of factor levels.

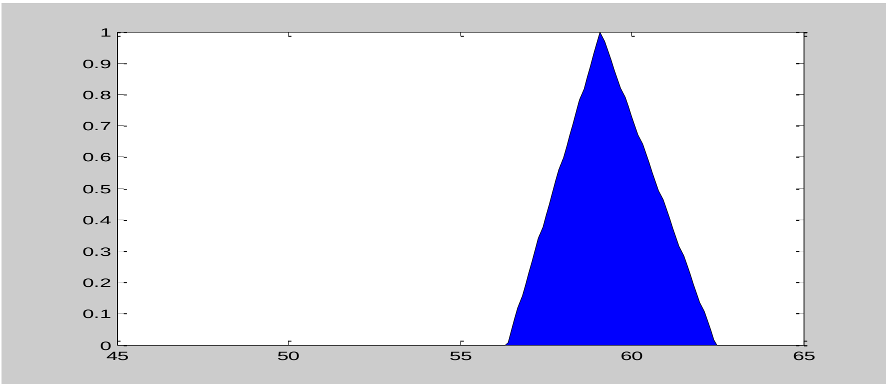


Figure 4.2: The MRR triangular fuzzy response at optimal combinations of factor levels.

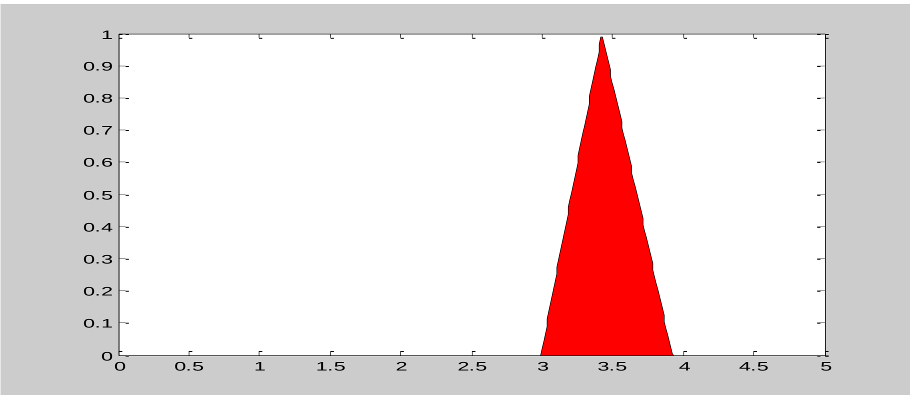


Figure 4.3: The SR fuzzy response at initial combinations of factor levels.

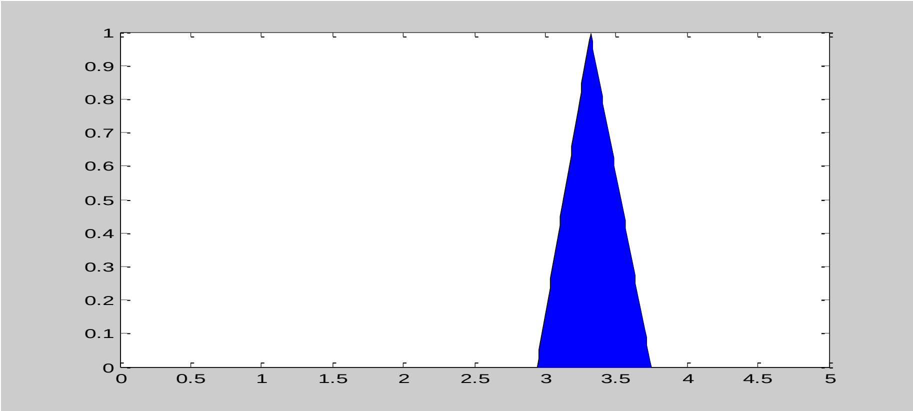


Figure 4.4: The SR fuzzy response at optimal combination of factor levels.

For the response MRR, the results are drawn from Figures 4.1 and 4.2, that the proposed approach provides a larger satisfactory region in the LTB range for the MRR response better than the initial condition, whereas the response SR, the result obtained from Figures 4.3 and 4.4 represents that the proposed approach provides a larger satisfactory region in the STB range, which is better than the initial condition.

The anticipated improvements in two responses due to setting factor levels at the optimal combination expressed in Table 4.9 are calculated and listed in Table 4.10.

Table 4.10: Anticipated improvement for the WEDM process.

	Initial fuzzy condition	Optimal fuzzy condition	Anticipated improvement
MRR (dB)	(49.39, 51.92, 54.58)	(56.38, 59.10, 62.46)	(6.99 , 7.18, 7.88)
SR (dB)	(2.99, 3.42, 3.92)	(2.94, 3.32, 3.75)	(-0.05, -0.1, -0.17)
Total anticipated improvement (dB)			**(6.94, 7.08, 7.71)**

Case II: Steps 4-9, are conducted for trapezoidal membership function, and are described as follows:

***Step* 4**: The trapezoidal fuzzy regression $\tilde{\eta}_1$ and $\tilde{\eta}_2$ are then obtained for the two responses and expressed as:

$\tilde{\eta}_1 =$ (30.6948, 30.7298, 30.7998, 30.8348) + (6.0766, 6.1507, 6.2989, 6.3730) X_1 + (-0.1854, - 0.1851, -0.1845, -0.18417) X_2 + (-0.0927, -0.0872, -0.0761, -0.0706) X_3 + (0.04512, 0.0497, 0.0587, 0.0633) X_4. (4.12)

and

$\tilde{\eta}_2 =$ (-8.5745, -8.5451,-8.4864, -8.4570) + (-2.2387, -2.0240, -1.5945, -1.3797) X_1 + (0.1606, 0.1883, 0.2436, 0.2713) X_2 + (-0.1083, -0.0995, -0.0821, -0.0734) X_3 + (-0.0974, -0.0876, -0.0681, -0.0583) X_4. (4.13)

Based on the results of step 2, the optimal fuzzy factor levels $\tilde{X}^*_{fj}$ that optimize MMR response are as follows:

$\tilde{X}^*_{11}$ = (1.2, 1.2, 1.2, 1.2), $\tilde{X}^*_{21}$ = (4, 4, 4, 4), $\tilde{X}^*_{31}$ = (8, 8, 8, 8), $\tilde{X}^*_{41}$ = (16, 16, 16, 16)

Whereas the optimal fuzzy factor levels $\tilde{X}^*_{fj}$ that optimize SR response are as follows:

$\tilde{X}^*_{12}$ = (0.6, 0.6, 0.6, 0.6), $\tilde{X}^*_{22}$ = (8, 8, 8, 8), $\tilde{X}^*_{32}$ = (8, 8, 8, 8), $\tilde{X}^*_{42}$ = (8, 8, 8, 8).

By substituting the optimal factor levels for the two responses in Eqs. (4.12) and (4.13), the values of the optimal trapezoidal fuzzy responses are respectively expressed as:

$\tilde{\eta}^*_1$ = (37.2267, 37.4678, 37.9509, 38.1936)

and

$\tilde{\eta}^*_2$ = (-10.2785, -9.7499, -8.6959, -8.1680)

***Step* 5**: The trapezoidal values of pay-off matrix $\tilde{\eta}_j(x)$ for MRR and SR responses are shown in Table 4.11.

Table 4.11: Pay-off matrix values for trapezoidal MRR and SR.

	$\tilde{\eta}_1(X)$	$\tilde{\eta}_2(X)$
$\tilde{X}_1^*$	(37.2267, 37.4678, 37.9509, 38.1936)	(-13.0433, -12.4183, -11.6230, -10.5474)
$\tilde{X}_2^*$	(32.4776, 32.6394, 32.9639, 33.1266)	(-10.2785, -9.7499, -8.6959, -8.1680)

***Step* 6**: The desirability function for each response is calculated using Eq. (3.6), where $\eta_{\min} = (30, 30, 30)$ and $\eta_{\max} = (40, 40, 40)$ are determine as an acceptable ranges for MRR response, while for SR response the acceptable ranges are $\eta_{\min} = (-13, -13, -13)$ and $\eta_{\max} = (-7, -7, -7)$. Then, the trapezoidal pay-off matrix for desirability values is shown in Table 4.12.

Table 4.12: Pay-off matrix for desirability values for MRR and SR.

	$\tilde{d}_1(\tilde{\eta}_1(X))$	$\tilde{d}_2(\tilde{\eta}_2(X))$
$\tilde{X}_1^*$	(0.7227, 0.7468, 0.7951, 0.8194)	(0, 0.097, 0.2295, 0.4088)
$\tilde{X}_2^*$	(0.2478, 0.2639, 0.2964, 0.3127)	(0.4536, 0.5417, 0.7174, 0.8052)

The following values from Table 4.12 are determined as:

$$\tilde{U}_1 = \tilde{d}_{11} = (0.7227, 0.7468, 0.7951, 0.8194)$$

$$\tilde{L}_1 = \min\left\{\tilde{d}_{11}, \tilde{d}_{21}\right\} = (0.2478, 0.2639, 0.2964, 0.3127)$$

and

$$\tilde{U}_2 = \tilde{d}_{22} = (0.4536, 0.5417, 0.7174, 0.8052)$$

$$\tilde{L}_2 = \min\left\{\tilde{d}_{12}, \tilde{d}_{22}\right\} = (0, 0.097, 0.2295, 0.4088)$$

***Step* 7**: Let $\lambda = 0.5,$ then the deviation function for each response is defined as:

$$D_j(X) = \frac{\eta_j^u(x) - \eta_j^c(x)}{(1-\lambda)} \qquad , j = 1, 2.$$

The fuzzy deviation functions for MRR and SR are respectively expressed as:

$$\tilde{D}_1(X) = 0.07 + 0.1482\tilde{X}_1 + 0.0006\tilde{X}_2 + 0.011\tilde{X}_3 + 0.0092\tilde{X}_4 \quad (4.14)$$

and

$$\tilde{D}_2(X) = 0.0588 + 0.4296\ \tilde{X}_1 + 0.0554\ \tilde{X}_2 + 0.0174\ \tilde{X}_3 + 0.0196\tilde{X}_4 \quad (4.15)$$

The pay-off matrix for deviation values is constituted as shown in Table 4.13.

Table 4.13: Pay-off matrix for deviation values for MRR and SR.

	$\tilde{D}_1(X)$	$\tilde{D}_2(X)$
$\tilde{X}_1^*$	(0.4854, 0.4854, 0.4854, 0.4854)	(1.2487, 1.2487, 1.2487, 1.2487)
$\tilde{X}_2^*$	(0.3253, 0.3253, 0.3253, 0.3253)	(1.0558, 1.0558, 1.0558, 1.0558)

From Table 4.13, the following values are expressed as:

$$\tilde{P}_1 = (0.4854,,0.4854,0.4854,0.4854),\ \tilde{Q}_1 = (0.4854,0.4854,0.4854,0.4854)$$

and

$$\tilde{P}_2 = (1.0558,1.0558,1.0558,1.0558),\ \ \tilde{Q}_2 = (1.2487,1.2487,1.2487,1.2487)$$

***Step* 8**: The final model is constructed as

$$\begin{aligned} &Max\left\{\tilde{d}_1(X),\tilde{d}_2(X)\right\} \\ &Min\left\{D_1(X),D_2(X)\right\} \\ &s.t \\ X &= \left\{\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{x}_4\right\} \in [\text{Factor Levels}] \end{aligned}$$

Based on results of steps 6 and 7, and using Eq. (3.9), and Eq. (3.10), the following constituted as:

$$\tilde{S}_1(X) = \begin{cases} 0, & \tilde{d}_1(X) \le \tilde{L}_1 \\ \dfrac{\tilde{d}_1(X) - (0.2478,0.2639,0.2964,0.3127)}{(0.7227,0.7468,0.7951,0.8194) - (0.2478,0.2639,0.2964,0.3127)}, & \tilde{L}_1 \le \tilde{d}_1 \le \tilde{U}_1 \\ 1, & \tilde{d}_1(X) \ge \tilde{U}_1 \end{cases}$$

$$\tilde{S}_2(X)=\begin{cases}0, & \tilde{d}_2(X)\le \tilde{L}_2\\ \dfrac{\tilde{d}_2(X)-(0,0.097,0.2295,0.4088)}{(0.4536,0.5417,0.7174,0.8052)-(0,0.097,0.2295,0.4088)}, & \tilde{L}_2\le \tilde{d}_2\le \tilde{U}_2\\ 1, & \tilde{d}_2(X)\ge \tilde{U}_2\end{cases}$$

and

$$\tilde{T}_1(X)=\begin{cases}1, & \tilde{D}_1(X)\le \tilde{P}_1\\ \dfrac{(0.4854,0.4854,0.4854,0.4854)-\tilde{D}_1(X)}{(0.4854,0.4854,0.4854,0.4854)-(0.4854,0.4854,0.4854,0.4854)}, & \tilde{P}_1\le \tilde{D}_1\le \tilde{Q}_1\\ 0, & \tilde{D}_1(X)\ge \tilde{Q}_1\end{cases}$$

$$\tilde{T}_2(X)=\begin{cases}1, & \tilde{D}_2(X)\le \tilde{P}_2\\ \dfrac{(1.2487,1.2487,1.2487,1.2487)-\tilde{D}_2(X)}{(1.2487,1.1687,1.1687,1.1687)-(1.0558,1.0558,1.0558,1.0558)}, & \tilde{P}_2\le \tilde{D}_2\le \tilde{Q}_2\\ 0, & \tilde{D}_2(X)\ge \tilde{Q}_2\end{cases}$$

Since it is desirable to maximize these two functions, the objective function is formulated as follows:

$$\begin{aligned}&Max\ \{\tilde{S}_1(X),\tilde{S}_2(X)\}\\ &Max\ \{\tilde{T}_1(X),\tilde{T}_2(X)\}\\ &s.t\\ &X=\{\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{x}_4\}\in[\text{Factor Levels}]\end{aligned}$$

By applying Zimmerman Max-Min operator, the two objectives is converted to single objective, this maximizes the minimum degree of satisfaction from the two objectives. Let $Min\ \tilde{S}_j(X)=\tilde{S}=(S^l,S^b,S^c,S^u)$, $Min\ \tilde{T}_j(X)=\tilde{T}=(T^l,T^b,T^c,T^u)$. Then, the problem is formulated as follows:

$$
\begin{aligned}
&Max \quad \tilde{S}(X) \\
&Max \quad \tilde{T}(X) \\
&s.t \\
&\tilde{d}_j(X) - \tilde{S}(\tilde{U}_j - \tilde{L}_j) \geq \tilde{L}_j \qquad , j = 1, 2. \\
&\tilde{D}_j(X) + \tilde{T}(\tilde{Q}_j - \tilde{P}_j) \leq \tilde{Q}_j \qquad , j = 1, 2. \\
&X = \{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4\} \in \text{[Factor Levels]}
\end{aligned}
$$

Finally, the final model using Eq. (3.13) is categorized to four types: model l, model b, model c, and model u, where models l and u as in triangular membership function, whereas the remaining models as shown below:

Model *b* :

$$
\begin{aligned}
&Max \quad 0.5 \times S^b + 0.5 \times T^b \\
&s.t \\
&0.07298 + 0.6151x_1^b - 0.01851x_2^b - 0.00872x_3^b + 0.00497x_4^b - 0.4829\, S^b \geq 0.2639 \\
&0.7425 - 0.3373x_1^b + 0.0314x_2^b - 0.01658x_3^b - 0.0146x_4^b - 0.4447\, S^b \geq 0.097 \\
&0.07000 + 0.1482x_1^b + 0.0006x_2^b + 0.011x_3^b + 0.0092x_4^b + 0\, T^b \leq 0.4854 \\
&0.0588 + 0.4296x_1^b + 0.0554x_2^b + 0.0174x_3^b + 0.0196x_4^b + 0.1929\, T^b \leq 1.2487 \\
&w_1 + w_2 = 1 \\
&0 \leq S^b, T^b \leq 1 \\
&X = \{x_1^b, x_2^b, x_3^b, x_4^b\} \in \text{[Factor Levels]}
\end{aligned}
$$

Model *c* :

$$
\begin{aligned}
&Max \quad 0.5 \times S^c + 0.5 \times T^c \\
&s.t \\
&0.08348 + 0.62989x_1^c - 0.01845x_2^c - 0.00761x_3^c + 0.00587x_4^c - 0.4987\, S^c \geq 0.2964 \\
&0.75227 - 0.26575x_1^c + 0.0406x_2^c - 0.01368x_3^c - 0.01135x_4^c - 0.4879\, S^c \geq 0.2295 \\
&0.0700 + 0.1482x_1^c + 0.0006x_2^c + 0.011x_3^c + 0.0092x_4^c + 0\, T^c \leq 0.4854 \\
&0.0588 + 0.4296x_1^c + 0.0554x_2^c + 0.0174x_3^c + 0.0196x_4^c + 0.1929\, T^c \leq 1.2487 \\
&w_1 + w_2 = 1 \\
&0 \leq S^c, T^c \leq 1 \\
&X = \{x_1^c, x_2^c, x_3^c, x_4^c\} \in \text{[Factor Levels]}
\end{aligned}
$$

***Step* 9**: The four models are solved, then the optimal factor levels of models l, b, c and u are shown in Table 4.14 as follows:

Table 4.14: The optimal factor levels of four models for WEDM process.

	Model l	Model b	Model c	Model u
X_1^*	0.9330	0.9477	0.9494	0.9624
X_2^*	5.1902	5.2911	5.3044	5.4184
X_3^*	8	8	8	8
X_4^*	8	8	8	8

Substituting the values the of factor levels from model *l*, model *b*, model *c*, and model *u*, respectively in Eq. (4.12), and in Eq. (4.13), the results are expressed as:

$\tilde{\eta}_1^*$ = (35.0220, 35.2953, 35.6621, 35.9117)

$\tilde{\eta}_2^*$ = (-11.4753, -10.9637, -9.9097, -9.3684)

By converting the $\tilde{\eta}_1^*$ to $\tilde{y}_1^*$ using Eq. (4.1) and converting $\tilde{\eta}_2^*$ to $\tilde{y}_2^*$ using Eq. (4.2), the optimal fuzzy responses are respectively obtained:

$\tilde{y}_1^*$ = (56.3767, 58.1788, 60.6884, 62.4576)

and

$\tilde{y}_2^*$ = (2.9405, 3.1296, 3.533, 3.7477)

The factor levels for initial condition were being set at the combinations $X_{1(2)}X_{2(2)}X_{3(2)}X_{4(2)}$. By substituting the values of factor levels for this combinations in Eq. (4.12), and in Eq. (4.13), and converting the results to $\tilde{y}_1$ and $\tilde{y}_2$ using Eq. (4.1) and Eq. (4.2), respectively, the obtained triangular fuzzy responses at initial combination of factor levels are written as:

$\tilde{y}_1$ = (49.3895, 50.6393, 53.2299, 54.5796)

and

$\tilde{y}_2 = (2.9910, 3.0001, 3.6642, 3.9221)$

Plotting the MRR triangular fuzzy response at initial combination of factor levels, $\tilde{y}_1$, and at optimal combination of factor levels, $\tilde{y}_1^*$, both are respectively represented in Figures 4.5 and 4.6. Similarly, plotting the SR triangular fuzzy response, $\tilde{y}_2$, at initial combination of factor levels, and at optimal combination of factor levels, $\tilde{y}_2^*$, are respectively represented in Figures 4.7, and 4.8.

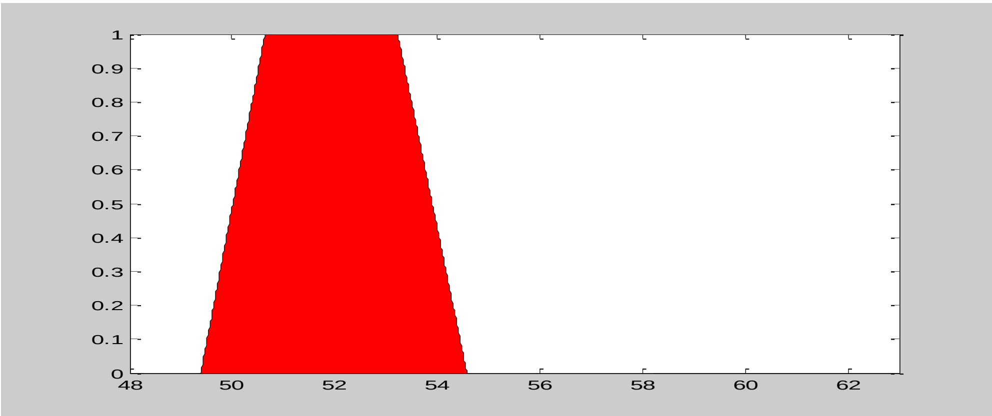


Figure 4.5: The MRR trapezoidal fuzzy response at initial combination of factor levels.

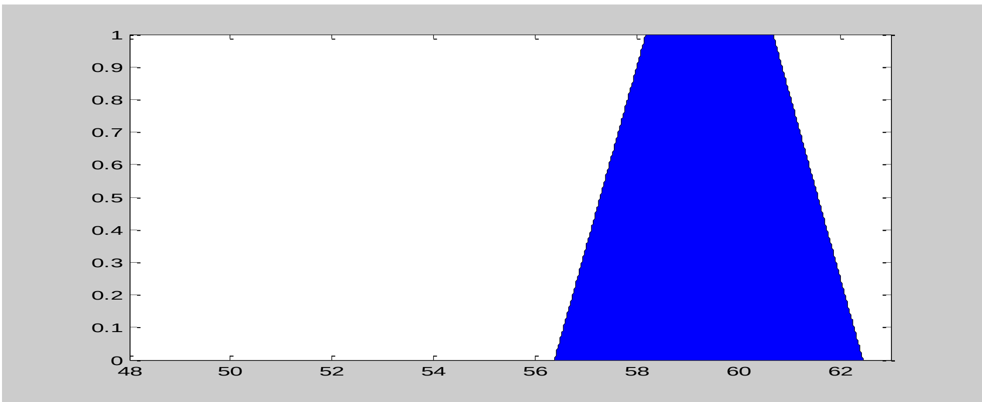


Figure 4.6: The MRR trapezoidal fuzzy response at optimal combination of factor levels.

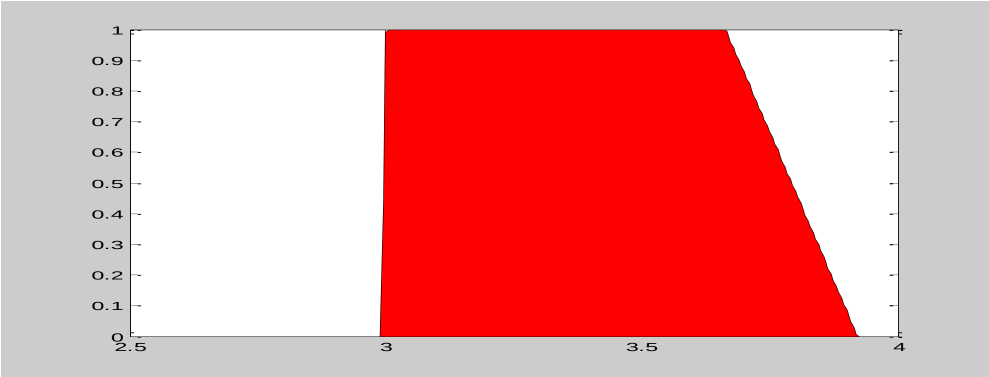


Figure 4.7: The SR trapezoidal fuzzy response at initial combination of factor levels.

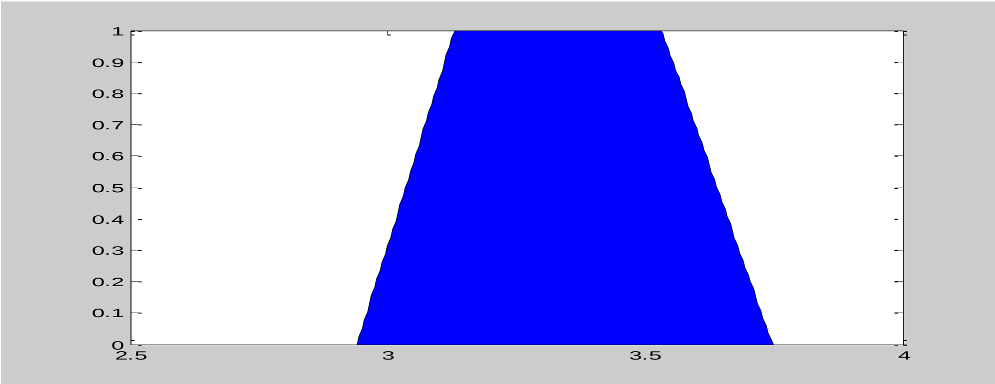


Figure 4.8: The SR trapezoidal fuzzy response at optimal combination of factor levels.

The anticipated improvements in the two responses due to setting factor levels at the optimal combination expressed in Table 4.14 are calculated and listed in Table 4.15.

Table 4.15: Anticipated improvement for the WEDM process.

	Initial fuzzy condition	Optimal fuzzy condition	Anticipated improvement
MRR (dB)	(49.39, 50.64, 53.23, 54.58)	(56.38, 58.18, 60.69, 62.46)	(6.99 , 7.54, 7.46, 7.88)
SR (dB)	(2.99, 3.00, 3.66, 3.92)	(2.94, 3.13, 3.53, 3.75)	(-0.05, 0.13, -0.13, -0.17)
Total anticipated improvement (dB)			**(6.94, 7.67, 7.33, 7.71)**

Comparing between triangular and trapezoidal membership functions, the following results are obtained:

(i). The proposed approach with each of the two membership functions provides a larger satisfactory region better than the initial condition.

(ii). The triangular membership function increases the satisfactory region, and represents the ideal (mean) solution as one value; however, trapezoidal membership function increases the flexibility of the fuzzy models, and represents the ideal (mean) solution as ranges. Table 4.16 the ideal solution for both MRR and SR.

Table 4.16: The ideal solutions for MRR and SR responses.

Responses	Triangular Ideal Solution	Trapezoidal Ideal Solution
MRR	(59.0956)	[58.1788-60.6884]
SR	(3.3184)	[3.1296-3.533]

(iii). Both membership functions deviation and fuzziness, and all inherent variability in the collected data have been minimized and optimized, by considering each replicate, in contrast with the traditional regression approaches.

4.2. Optimization of the sputtering process parameter

Chen *et. al* (2009) optimized the sputtering process parameters of GZO films using the grey–Taguchi method. The factors studied were (X_1) R.F. power, (X_2) sputtering pressure, (X_3) deposition time, (X_4) substrate temperature, and (X_5) post-annealing temperature. Three responses were of main interest including deposition rate (DR) electrical resistively (ER) and optical transmittance (OT), where (y_1, LTB), (y_2, STB), and (y_3, LTB). The L_{18} array shown in Table 4.17 was utilized in experimental work, and factors and their levels for experiment clarified in Table 4.18.

Table 4.17: Orthogonal array and the experimental data for the sputtering process.

Exp. i	Factor levels					DR		ER		OT	
	X_1	X_2	X_3	X_4	X_5	y_{i11}	y_{i12}	y_{i21}	y_{i22}	y_{i31}	y_{i32}
1	1	1	1	1	1	4.5	4.7	14.9	15.3	88.4	88.4
2	1	2	2	2	2	5.6	5.6	9.8	9.7	87.7	87.7
3	1	3	3	3	3	5.0	4.9	7.9	7.8	88.1	88.1
4	2	1	1	2	2	9.6	9.3	5.4	5.6	89.2	89.3
5	2	2	2	3	3	11.1	11.3	4.6	4.3	87.1	87.0
6	2	3	3	1	1	10.0	10.0	6.5	6.6	84.7	84.7
7	3	1	2	1	3	19.9	20.2	1.6	1.7	86.6	86.6
8	3	2	3	2	1	21.6	21.6	1.9	2.0	82.3	82.4
9	3	3	1	3	2	20.9	20.9	1.8	1.6	85.6	85.3
10	1	1	3	3	2	4.8	4.6	7.3	7.0	87.6	87.6
11	1	2	1	1	3	5.0	4.9	6.9	7.1	89.1	89.1
12	1	3	2	2	1	4.9	4.8	7.8	7.7	87.4	87.4
13	2	1	2	3	1	9.7	9.7	6.1	5.9	87.0	87.0
14	2	2	3	1	2	11.1	11.6	6.0	5.8	83.7	83.7
15	2	3	1	2	3	10.7	10.8	5.5	5.7	88.4	88.3
16	3	1	3	2	3	19.5	19.4	1.0	1.1	83.1	83.1
17	3	2	1	3	1	22.1	22.0	1.2	1.3	85.7	85.7
18	3	3	2	1	2	20.5	20.5	1.4	1.3	83.9	83.7

Table 4.18: Factors and their levels for sputtering experiment.

Factor	Process variables	Level 1	Level 2	Level 3
X_1	R.F. power (W)	50	100	200
X_2	Sputtering pressure (Pa)	0.13	0.67	1.33
X_3	Deposition time (min)	30	60	90
X_4	Substrate temperature (8C)	25	50	100
X_5	Post-annealing temperature (8C)	0	100	200

Let η_{i1r}, η_{i2r}, and η_{i3r} denote the S/N ratio for DR, ER, and OT responses at experiment i (i =1,…, 18) with replicate r (r =1, 2), respectively. Then, the η_{i1r}, η_{i2r}, and η_{i3r} are respectively calculated for each replicate as follows:

$$\eta_{i1r} = -10\log_{10}(1/y_{i1r}^2) \qquad i=1,...,18 \quad r=1,2 \text{ for DR} \qquad (4.16)$$

$$\eta_{i2r} = -10\log_{10}(y_{i2r}^2) \qquad i=1,...,18 \quad r=1,2 \text{ for ER} \qquad (4.17)$$

$$\eta_{i3r} = -10\log_{10}(1/y_{i3r}^2) \qquad i=1,...,18 \quad r=1,2 \text{ for OT} \qquad (4.18)$$

The obtained η_{i1r}, η_{i2r}, and η_{i3r} for each replicate are summarized in Table 4.19.

Table 4.19: The experimental results for each replicate for sputtering experiment.

Exp. i	DR		ER		OT	
	$\eta_{i\,11}$	$\eta_{i\,12}$	$\eta_{i\,21}$	$\eta_{i\,22}$	$\eta_{i\,31}$	$\eta_{i\,32}$
1	13.064	13.442	-23.464	-23.694	38.929	38.929
2	14.964	14.964	-19.825	-19.735	38.860	38.860
3	13.979	13.804	-17.953	-17.842	38.900	38.900
4	19.645	19.370	-14.648	-14.964	39.007	39.017
5	20.906	21.062	-13.255	-12.669	38.800	38.790
6	20.000	20.000	-16.258	-16.391	38.558	38.558
7	25.977	26.107	-4.0824	-4.609	38.750	38.750
8	26.689	26.690	-5.5751	-6.021	38.308	38.319
9	26.403	26.403	-5.1055	-4.082	38.649	38.619
10	13.625	13.255	-17.266	-16.902	38.850	38.850
11	13.979	13.804	-16.777	-17.025	38.998	38.998
12	13.804	13.625	-17.842	-17.730	38.830	38.830
13	19.735	19.735	-15.707	-15.417	38.790	38.7900
14	20.906	21.289	-15.563	-15.269	38.455	38.455
15	20.588	20.668	-14.807	-15.117	38.929	38.919
16	25.801	25.753	0.000	-0.828	38.392	38.392
17	26.888	26.848	-1.584	-2.279	38.660	38.600
18	26.235	26.235	-2.923	-2.279	38.475	38.455

The $\bar{\eta}_{lfr}$ values are calculated for all factor levels then displayed in Table 4.20 for the DR response, whereas Table 4.21 for ER, finally Table 4.22 for OT response.

Table 4.20: The optimal factor levels for the two replicates of DR.

Factor (f)	Replicate r=1			Replicate r=2		
	Level 1	Level 2	Level 3	Level 1	Level 2	Level 3
X_1	16.002	20.297	26.332	13.816	20.354	26.340
X_2	19.641	20.722	20.168	19.611	20.776	20.123
X_3	20.095	20.270	20.167	20.089	20.288	20.132
X_4	20.027	20.248	20.256	20.146	20.179	20.185
X_5	20.030	20.296	20.205	20.057	20.253	20.200

Table 4.21: The optimal factor levels for the first and second replicates of ER.

Factor (f)	Replicate r=1			Replicate r=2		
	Level 1	Level 2	Level 3	Level 1	Level 2	Level 3
X_1	-18.854	-15.040	-3.212	-18.821	-14.971	-3.350
X_2	-12.528	-12.096	-12.481	-12.736	-12.166	-12.240
X_3	-12.731	-12.272	-12.103	-12.86	-12.073	-12.209
X_4	-13.178	-12.116	-11.812	-13.211	-12.399	-11.532
X_5	-13.405	-12.555	-11.146	-13.589	-12.205	-11.348

Table 4.22: The optimal factor levels for the first and second replicate of OT.

Factor (f)	**Replicate** r=1			**Replicate** r=2		
	Level 1	Level 2	Level 3	Level 1	Level 2	Level 3
X_1	38.894	38.757	38.539	38.894	38.755	38.532
X_2	38.787	38.680	38.724	38.788	38.680	38.713
X_3	38.862	38.751	38.577	38.857	38.746	38.579
X_4	38.694	38.721	38.775	38.691	38.723	38.768
X_5	38.679	38.716	38.795	38.681	38.709	38.792

For DR, the combination of optimal factor levels for the two replicates are identified as $X1_{(3)}\ X2_{(2)}\ X3_{(2)}\ X4_{(3)}\ X5_{(2)}$, and for ER, the combination of optimal factor levels for the first and second replicates of ER response are respectively identified as $X1_{(3)}\ X2_{(2)}\ X3_{(3)}\ X4_{(3)}\ X5_{(3)}$, $X1_{(3)}X2_{(2)}X3_{(2)}X4_{(3)}X5_{(3)}$, whereas for OT, the combination of optimal factor levels for the two replicates is identified as $X1_{(1)}X2_{(1)}X3_{(1)}X4_{(3)}X5_{(3)}$.

The multiple linear regression equations for η_{11} and η_{12} are written respectively as:

$$\eta_{11} = 10.293 + 0.07965X_1 + 0.3923X_2 + 1.2\times10^{-3}X_3 + 2.64\times10^{-3}X_4 + 8.75\times10^{-4}X_5 \quad (4.19.a)$$

R-Sq (adj) = 93%

$$\eta_{12} = 10.415 + 0.08012X_1 + 0.3746X_2 + 7.2\times10^{-4}X_3 + 4.6\times10^{-4}X_4 + 7.18\times10^{-4}X_5 \quad (4.19.b)$$

R-Sq (adj) = 92%

Whereas, the multiple linear regression equations for η_{21} and η_{22} are respectively represented by:

$$\eta_{21} = -27.499 + 0.1063X_1 + 0.016X_2 + 0.0105X_3 + 0.0165X_4 + 0.0113X_5 \quad (4.20.a)$$

R-Sq (adj) = 92.4%

$$\eta_{22} = -27.947 + 0.105X_1 + 0.394X_2 + 0.0109X_3 + 0.0217X_4 + 0.0112X_5 \quad (4.20.b)$$

R-Sq (adj) = 93.1%

Finally, the multiple linear regression equations for η_{31} and η_{32} equations are respectively expressed as:

$$\eta_{31} = 39.202 - 2.34\times10^{-3} X_1 - 0.048 X_2 - 4.75\times10^{-3} X_3 + 1.11\times10^{-3} X_4 + 5.78\times10^{-4} X_5 \qquad (4.21.a)$$

R-Sq (adj) = 84.9%

$$\eta_{32} = 39.211 - 2.39\times10^{-3} X_1 - 0.058 X_2 - 4.64\times10^{-3} X_3 + 1.02\times10^{-3} X_4 + 5.53\times10^{-4} X_5 \qquad (4.21.b)$$

R-Sq (adj) = 84%

Case I: Steps 4-9, are conducted for triangular membership function, and are described as follow:

The triangular fuzzy regression $\tilde{\eta}_1$, $\tilde{\eta}_2$ and $\tilde{\eta}_3$ are obtained for the three responses and expressed as:

$\tilde{\eta}_1 =$ (10.2677, 10.354, 10.4403) + (0.0796, 0.0799, 0.08022) X_1 + (0.3709, 0.3835, 0.396) X_2 + (0.0006, 0.001, 0.0013) X_3 + (0.00001, 0.0016, 0.0031) X_4 + (0.0007, 0.0008, 0.0009) X_5 (4.22)

$\tilde{\eta}_2 =$ (-28.0398, -27.723, -27.4062) + (0.1047, 0.1056, 0.1065) X_1 + (-0.0622, 0.2048, 0.4718) X_2 + (0.0104, 0.01067, 0.0109) X_3 + (0.0154, 0.0191, 0.0227) X_4 + (0.0112, 0.01125, 0.0113) X_5 (4.23)

$\tilde{\eta}_3 =$ (39.1999, 39.2062, 39.2124) + (-0.00239, -0.00236, -0.00233) X_1 + (-0.0603, -0.0532, -0.0461) X_2 + (-0.0048, -0.0047, -0.0046) X_3 + (0.001, 0.00105, 0.00109) X_4 + (0.00058, 0.00057, 0.00055) X_5 (4.24)

Based on the results of step 2, the optimal fuzzy factor levels $\tilde{X}^*_{fj}$ that optimize DR are as follows:

$\tilde{X}^*_{11}$ = (200, 200, 200), $\tilde{X}^*_{21}$ = (0.67, 0.67, 0.67), $\tilde{X}^*_{31}$ = (60, 60, 60), $\tilde{X}^*_{41}$ = (100, 100, 100), $\tilde{X}^*_{51}$ = (100, 100, 100)

Whereas, the optimal fuzzy factor levels $\tilde{X}^*_{fj}$ that optimize ER are as follows:

$\tilde{X}^*_{12}$ = (200, 200, 200), $\tilde{X}^*_{22}$ = (0.67, 0.67, 0.67), $\tilde{X}^*_{32}$ = (53.7868, 75, 96.2132), $\tilde{X}^*_{42}$ = (100, 100, 100), $\tilde{X}^*_{51}$ = (200, 200, 200)

Finally, the optimal fuzzy factor levels $\tilde{X}_{fj}^{*}$ that optimize OT are as follows:

$\tilde{X}_{13}^{*}$ = (50, 50, 50), $\tilde{X}_{23}^{*}$ = (0.13, 0.13, 0.13), $\tilde{X}_{33}^{*}$ = (30, 30, 30), $\tilde{X}_{43}^{*}$ = (100, 100, 100), $\tilde{X}_{53}^{*}$ = (200, 200, 200).

By substituting the optimal factor levels for the three responses in Eqs. (4.22) to (4.24), respectively, the values of the optimal triangular fuzzy responses are respectively expressed as:

$\tilde{\eta}_1^{*}$ = (26.5432, 26.8909, 27.2276)

$\tilde{\eta}_2^{*}$ = (-2.8021, -1.5055, -0.2114)

and

$\tilde{\eta}_3^{*}$ = (39.1386, 39.1593, 39.1769)

The values of pay-off matrix $\tilde{\eta}_j(x)$ for DR, ER, and OT are shown in Table 4.23.

Table 4.23: Pay-off matrix values for DR, ER, and OT.

	$\tilde{\eta}_1(X)$	$\tilde{\eta}_2(X)$	$\tilde{\eta}_3(X)$
$\tilde{X}_1^{*}$	(26.5432, 26.8909, 27.2276)	(-3.8575, -2.7906, -1.7361)	(38.5485, 38.5786, 38.6065)
$\tilde{X}_2^{*}$	(26.6095, 26.9859, 27.3647)	(-2.8021, -1.5055, -0.2114)	(38.4979, 38.4956, 38.6333)
$\tilde{X}_3^{*}$	(14.4549, 14.7489, 15.0318)	(-18.7209, -17.9263, -17.1629)	(39.1386, 39.1593, 39.1769)

The desirability function for each response is calculated using Eq. (3.6), where $\eta_{\min} = (12,12,12)$ and $\eta_{\max} = (30,30,30)$ are determine as an acceptable ranges for DR, while for ER, the acceptable ranges are $\eta_{\min} = (-24,-24,-24)$ and $\eta_{\max} = (0,0,0)$,

finally, the acceptable ranges for OT are $\eta_{\min} = (38, 38, 38)$ and $\eta_{\max} = (40, 40, 40)$. Then, the pay-off matrix for desirability values is shown in Table 4.24.

Table 4.24: Pay-off matrix for desirability values for DR, ER, and OT.

	$\tilde{d}_{j1}(\tilde{\eta}_{j1}(X))$	$\tilde{d}_{j2}(\tilde{\eta}_{j2}(X))$	$\tilde{d}_{j3}(\tilde{\eta}_{j3}(X))$
$\tilde{X}_1^*$	(0.808, 0.8273, 0.846)	(0.8393, 0.8837, 0.9277)	(0.2743, 0.2893, 0.3033)
$\tilde{X}_2^*$	(0.812,0.8326, 0.8536)	(0.8832, 0.9373, 0.9912)	(0.2490, 0.2826, 0.3167)
$\tilde{X}_3^*$	(0.136, 0.1527, 0.1684)	(0.2200, 0.2526, 0.2849)	(0.5693, 0.5797, 0.5885)

From Table 4.24, the following values are determined:

$$\tilde{U}_1 = \tilde{d}_{11} = (0.8080, 0.8273, 0.8460),\ \tilde{L}_1 = (0.1364, 0.1527, 0.1684)$$

$$\tilde{U}_2 = \tilde{d}_{22} = (0.8832, 0.9373, 0.9912),\ \tilde{L}_2 = (0.2200, 0.2526, 0.2849)$$

$$\tilde{U}_3 = \tilde{d}_{33} = (0.5693, 0.5797, 0.5885),\ \tilde{L}_3 = (0.249, 0.2826, 0.3167)$$

The fuzzy deviation functions for DR, ER, and OT are expressed respectively as:

$$\tilde{D}_1 = 0.0863 + 0.00044\tilde{X}_1 + 0.0126\tilde{X}_2 + 0.0004\tilde{X}_3 + 0.0016\tilde{X}_4 + 0.0001\tilde{X}_5 \quad (4.25)$$

$$\tilde{D}_2 = 0.3168 + 0.0008\ \tilde{X}_1 + 0.267\ \tilde{X}_2 + 0.0002\ \tilde{X}_3 + 0.0036\ \tilde{X}_4 + 0.00004\tilde{X}_5 \quad (4.26)$$

$$\tilde{D}_3 = 0.0062 + 0.00004\tilde{X}_1 + 0.007\tilde{X}_2 + 0.0001\tilde{X}_3 + 0.00004\tilde{X}_4 + 0.00001\tilde{X}_5 \quad (4.27)$$

The pay-off matrix for deviation values is constituted in Table 4.25.

Table 4.25: Pay-off matrix for deviation values for DR, ER, and OT.

	$\tilde{D}_1(X)$	$\tilde{D}_2(X)$	$\tilde{D}_3(X)$
$\tilde{X}_1^*$	(0.3768, 0.3768, 0.3768)	(1.0317, 1.0317, 1.0317)	(0.0300, 0.0300, 0.0300)
$\tilde{X}_2^*$	(0.3844, 0.3929, 0.4013)	(1.0344, 1.0387, 1.0429)	(0.0303, 0.0324, 0.0345)
$\tilde{X}_3^*$	(0.3020, 0.3020, 0.3020)	(0.7655, 0.7655, 0.7655)	(0.0181, 0.0181, 0.0181)

From Table 4.25, the following values are expressed as:

$$\tilde{P}_1 = (0.3768, 0.3768, 0.3768),\ \tilde{Q}_1 = (0.3844, 0.3929, 0.4013)$$

$\tilde{P}_2 = (1.0344, 1.0387, 1.0429),\ \tilde{Q}_2 = (1.0344, 1.0387, 1.0429)$

$\tilde{P}_3 = (0.0181, 0.0181, 0.0181),\ \tilde{Q}_3 = (0.0303, 0.0324, 0.0345)$

The final model is constructed as:

$$
\begin{aligned}
& Max\ \left\{\tilde{d}_1(X), \tilde{d}_2(X), \tilde{d}_3(X)\right\} \\
& Min\ \left\{D_1(X), D_2(X), D_3(X)\right\} \\
& s.t \\
& X = \left\{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5\right\} \in [\text{Factor Levels}]
\end{aligned}
$$

Based on the results of steps 6 and 7, and using Eqs. (3.9) and (3.10), respectively, the followings are constituted as:

$$
\tilde{S}_1(X) = \begin{cases} 0, & \tilde{d}_1(X) \le \tilde{L}_1 \\ \dfrac{\tilde{d}_1(x) - (0.1364, 0.1527, 0.1684)}{(0.808, 0.8273, 0.846) - (0.1364, 0.1527, 0.1684)}, & \tilde{L}_1 \le \tilde{d}_1 \le \tilde{U}_1 \\ 1, & \tilde{d}_1(X) \ge \tilde{U}_1 \end{cases}
$$

$$
\tilde{S}_2(X) = \begin{cases} 0, & \tilde{d}_2(X) \le \hat{L}_2 \\ \dfrac{\tilde{d}_2(X) - (0.2200, 0.2526, 0.2849)}{(0.8832, 0.9373, 0.9912) - (0.2200, 0.2526, 0.2849)}, & \hat{L}_2 \le \tilde{d}_2 \le \tilde{U}_2 \\ 1, & \tilde{d}_2(X) \ge \tilde{U}_2 \end{cases}
$$

$$
\tilde{S}_3(X) = \begin{cases} 0, & \tilde{d}_3(X) \le \tilde{L}_3 \\ \dfrac{\tilde{d}_3(X) - (0.249, 0.2826, 0.3167)}{(0.5693, 0.5797, 0.5885) - (0.249, 0.2826, 0.3167)}, & \tilde{L}_3 \le \tilde{d}_3 \le \tilde{U}_3 \\ 1, & \tilde{d}_3(X) \ge \tilde{U}_3 \end{cases}
$$

and

$$
\tilde{T}_1(X) = \begin{cases} 1, & \tilde{D}_1(X) \le \tilde{P}_1 \\ \dfrac{(0.3844, 0.3929, 0.4013) - \tilde{D}_1(x)}{(0.3844, 0.3929, 0.4013) - (0.3768, 0.3768, 0.3768)}, & \tilde{P}_1 \le \tilde{D}_1 \le \tilde{Q}_1 \\ 0, & \tilde{D}_1(X) \ge \tilde{Q}_1 \end{cases}
$$

$$\tilde{T}_2(X) = \begin{cases} 1, & \tilde{D}_2(X) \le \tilde{P} \\ \dfrac{(1.0344,1.0387,1.0429) - \tilde{D}_2(x)}{(1.0344,1.0387,1.0429) - (1.0344,1.0387,1.0429)}, & \tilde{P}_2 \le \tilde{D}_2 \le \tilde{Q}_2 \\ 0, & \tilde{D}_2(X) \ge \tilde{Q}_2 \end{cases}$$

$$\tilde{T}_3(X) = \begin{cases} 1, & \tilde{D}_3(X) \le \tilde{P}_3 \\ \dfrac{(0.0303,0.0324,0.0345) - \tilde{D}_3(x)}{(0.0303,0.0324,0.0345) - (0.0181,0.0181,0.0181)}, & \tilde{P}_3 \le \tilde{D}_3 \le \tilde{Q}_3 \\ 0, & \tilde{D}_3(X) \ge \tilde{Q}_3 \end{cases}$$

Since it is desirable to maximize these two functions, the objective function is formulated as follows:

$$\begin{aligned} &Max \left\{\tilde{S}_1(X), \tilde{S}_2(X), \tilde{S}_3(X)\right\} \\ &Max \left\{\tilde{T}_1(X), \tilde{T}_2(X), \tilde{T}_3(X)\right\} \\ &s.t \\ &X = \left\{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5\right\} \in \text{[Factor Levels]} \end{aligned}$$

By applying Zimmerman Max-Min operator, the two objectives is converted to single objective, this maximizes the minimum degree of satisfaction from the two objectives. Let $Min \quad \tilde{S}_j(x) = \tilde{S} = (S^l, S^m, S^u)$, $Min \quad \tilde{T}_j(x) = \tilde{T} = (T^l, T^m, T^u)$. Then, the problem is formulated as follows:

$$\begin{aligned} &Max \quad \tilde{S}(X) \\ &Max \quad \tilde{T}(X) \\ &s.t \\ &\tilde{d}_j(X) - \tilde{S}(\tilde{U}_j - \tilde{L}_j) \ge \tilde{L}_j \qquad , j = 1,2. \\ &\tilde{D}_j(X) + \tilde{T}(\tilde{Q}_j - \tilde{P}_j) \le \tilde{Q}_j \qquad , j = 1,2. \\ &X = \left\{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4\right\} \in \text{[Factor Levels]} \end{aligned}$$

Finally, the final model using Eq. (3.13) is categorized to three types: model l, model m, model u as shown below:

Model l **:**

$$
\begin{aligned}
&Max \quad 0.5\times S^{l}+0.5\times T^{l}\\
&s.t\\
&-0.09624+0.00442x_1^{l}+0.0206x_2^{l}+3.33\times10^{-5}x_3^{l}+5.6\times10^{-7}x_4^{l}+3.9\times10^{-5}x_5^{l}-0.6716S^{l}\geq 0.1364\\
&-0.1683+0.00436x_1^{l}-0.0026x_2^{l}+4.33\times10^{-4}x_3^{l}+6.42\times10^{-4}x_4^{l}+4.67\times10^{-4}x_5^{l}-0.6632S^{l}\geq 0.22\\
&0.59995-0.001195x_1^{l}-0.03015x_2^{l}-0.0024x_3^{l}+0.0005x_4^{l}+2.75\times10^{-4}x_5^{l}-0.3203S^{l}\geq 0.2490\\
&0.0863+0.00044x_1^{l}+0.126x_2^{l}+0.0004x_3^{l}+0.0016x_4^{l}+0.0001x_5^{l}+0.0076T^{l}\leq 0.3844\\
&0.3168+0.0008x_1^{l}+0.267x_2^{l}+0.0002x_3^{l}+0.0036x_4^{l}+0.00004x_5^{l}+0T^{l}\leq 1.0344\\
&0.0062+0.00004x_1^{l}+0.007x_2^{l}+0.0001x_3^{l}+0.00004x_4^{l}+0.00001x_5^{l}+0.0122T^{l}\leq 0.0303\\
&w_1+w_2=1\\
&0\leq S^{l},T^{l}\leq 1\\
&X=\left\{x_1^{l},x_2^{l},x_3^{l},x_4^{l},x_5^{l}\right\}\in[\text{Factor Levels}]
\end{aligned}
$$

Model m **:**

$$
\begin{aligned}
&Max \quad 0.5\times S^{m}+0.5\times T^{m}\\
&s.t\\
&-0.09144+0.004439x_1^{m}+0.02131x_2^{m}+5.56\times10^{-5}x_3^{m}+8.89\times10^{-5}x_4^{m}+4.44\times10^{-5}x_5^{m}-0.6589S^{m}\geq 0.1527\\
&-0.1551+0.0044x_1^{m}+0.008533x_2^{m}+4.46\times10^{-4}x_3^{m}+7.96\times10^{-4}x_4^{m}+4.69\times10^{-4}x_5^{m}-0.6847S^{m}\geq 0.2526\\
&0.6031-0.00118x_1^{m}-0.0266x_2^{m}-0.00235x_3^{m}+5.25\times10^{-4}x_4^{m}+2.85\times10^{-4}x_5^{m}-0.2971S^{m}\geq 0.3929\\
&0.0863+0.00044x_1^{m}+0.126x_2^{m}+0.0004x_3^{m}+0.0016x_4^{m}+0.0001x_5^{m}+0.0161T^{m}\leq 0.3844\\
&0.3168+0.0008x_1^{m}+0.267x_2^{m}+0.0002x_3^{m}+0.0036x_4^{m}+0.00004x_5^{m}+0T^{m}\leq 1.0387\\
&0.0062+0.00004x_1^{m}+0.007x_2^{m}+0.0001x_3^{m}+0.00004x_4^{m}+0.00001x_5^{m}+0.0143T^{m}\leq 0.0324\\
&w_1+w_2=1\\
&0\leq S^{m},T^{m}\leq 1\\
&X=\left\{x_1^{m},x_2^{m},x_3^{m},x_4^{m},x_5^{m}\right\}\in[\text{Factor Levels}]
\end{aligned}
$$

Model u :

$$
\begin{aligned}
&Max \quad 0.5\times S^{u}+0.5\times T^{u}\\
&s.t\\
&\text{-}0.0867+0.0045x_1^{u}+0.022x_2^{u}+7.22\times10^{-5}x_3^{u}+1.72\times10^{-4}x_4^{u}+5\times10^{-5}x_5^{u}-0.6776S^{u}\geq 0.1684\\
&\text{-}0.1419+0.0044x_1^{u}+0.0197x_2^{u}+4.5\times10^{-4}x_3^{u}+9.46\times10^{-4}x_4^{u}+4.71\times10^{-4}x_5^{u}-0.7063S^{u}\geq 0.2849\\
&0.6062-0.00117x_1^{u}-0.0231x_2^{u}-0.0023x_3^{u}+5.45\times10^{-4}x_4^{u}+2.9\times10^{-4}x_5^{u}-0.2718S^{u}\geq 0.3167\\
&0.0864+0.00044x_1^{u}+0.126x_2^{u}+0.0004x_3^{u}+0.0016x_4^{u}+0.0001x_5^{u}+0.0245T^{u}\leq 0.4013\\
&0.3168+0.0008x_1^{u}+0.267x_2^{u}+0.0002x_3^{u}+0.0036x_4^{u}+0.00004x_5^{u}+0T^{u}\leq 1.0429\\
&0.0062+0.00004x_1^{u}+0.0071x_2^{u}+0.0001x_3^{u}+0.00004x_4^{u}+0.00001x_5^{u}+0.0164T^{u}\leq 0.0345\\
&w_1+w_2=1\\
&0\leq S^{u},T^{u}\leq 1\\
&X=\left\{x_1^{u},x_2^{u},x_3^{u},x_4^{u},x_5^{u}\right\}\in[\text{Factor Levels}]
\end{aligned}
$$

The three models are solved, then the optimal factor levels of models l, m, and u are determined as shown in Table 4.26 as follows:

Table 4.26: The optimal factor levels of three models for sputtering process.

	Model *l*	Model *m*	Model *u*
x_1^*	107.1020	139.8411	200
x_2^*	0.13	0.13	0.13
x_3^*	30	30	30
x_4^*	25	25	42.6490
x_5^*	0	200	200

By substituting the values of factor levels from model *l*, model *m*, and model *u*, in Eqs. (4.22) to (4.24), the results are respectively expressed as:

$\tilde{\eta}_1^*$ = (18.8595, 21.7728, 26.8870)

$\tilde{\eta}_2^*$ = (-16.1373, -9.8816, -2.4897)

and

$\tilde{\eta}_3^*$ = (38.7649, 38.8171, 38.8685)

By converting the $\tilde{\eta}_1^*$ to $\tilde{y}_1^*$, $\tilde{\eta}_2^*$ to $\tilde{y}_2^*$, and $\tilde{\eta}_3^*$ to $\tilde{y}_3^*$ using Eqs. (4.16) to (4.18), the optimal fuzzy responses are respectively obtained as:

$\tilde{y}_1^*$ = (8.7695, 12.2642, 22.0978)

$\tilde{y}_2^*$ = (1.3319, 3.1195, 6.0101)

$\tilde{y}_3^*$ = (86.7451, 87.2680, 87.7859)

The factor levels for initial condition were set at the combination $X1_{(2)}X2_{(2)}X3_{(1)}X4_{(3)}X5_{(1)}$. By substituting the factor levels for this combination in Eqs. (4.22) to (4.24), respectively, then converting the results to $\tilde{y}_1$, $\tilde{y}_2$, and $\tilde{y}_3$ using Eqs. (4.16) to (4.18), respectively, the obtained fuzzy responses at initial combination of factor levels are written as:

$\tilde{y}_1$ = (8.4180, 8.7007, 8.9915)

$\tilde{y}_2$ = (4.9222, 5.4927, 6.1372)

$\tilde{y}_3$ = (87.8668, 88.0907, 88.3029)

Plotting the DR fuzzy response at initial combination of factor levels, $\tilde{y}_1$, and at optimal combination of factor levels, $\tilde{y}_1^*$, is represented in Figures 4.9 and 4.10, respectively. Similarly, plotting ER fuzzy response $\tilde{y}_2 / \tilde{y}_2^*$ at initial/ optimal combination of factor levels are represented in Figures 4.11, 4.12, respectively, finally plotting OT fuzzy response $\tilde{y}_3 / \tilde{y}_3^*$ at initial/ optimal combination of factor levels are shown in Figures 4.13, and 4.14, respectively.

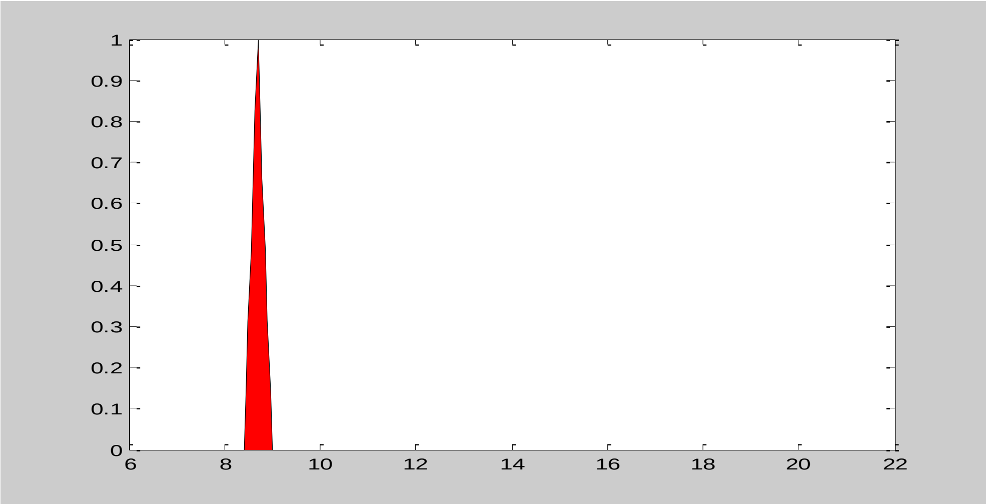


Figure 4.9: The DR fuzzy response at initial combination of factor levels.

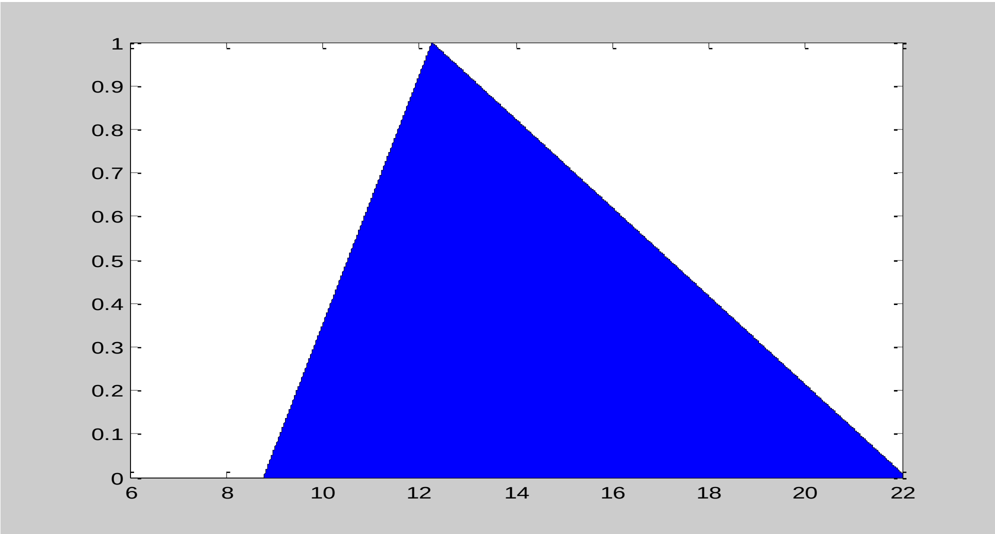


Figure 4.10: The DR fuzzy response at optimal combination of factor levels.

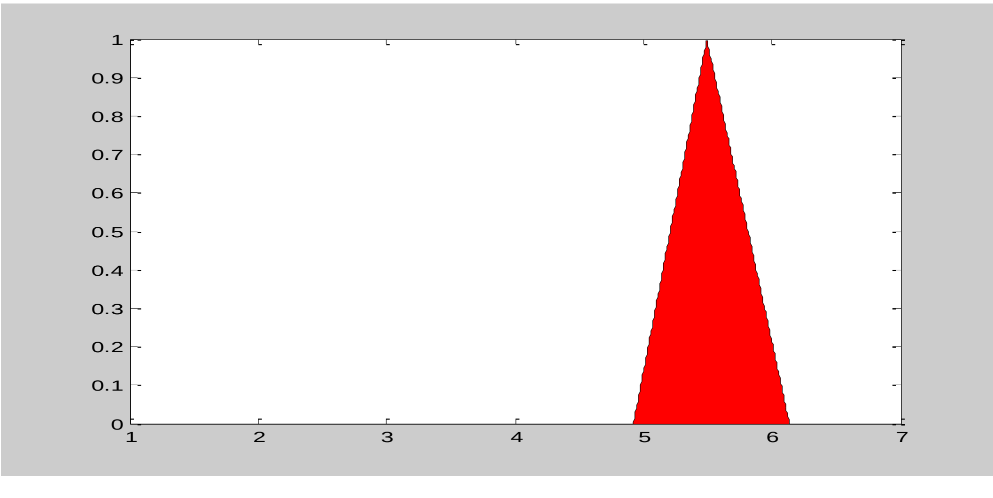


Figure 4.11: The ER fuzzy response at initial combination of factor levels.

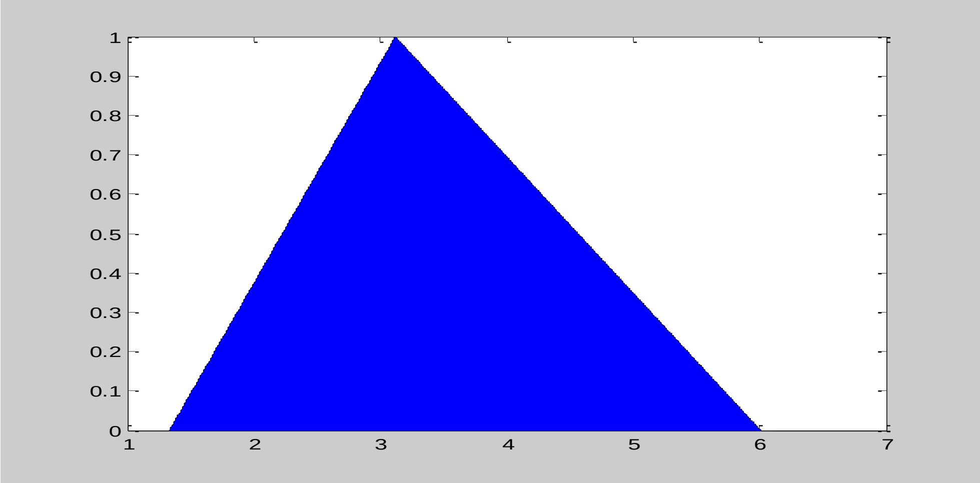


Figure 4.12: The ER fuzzy response at optimal combination of factor levels.

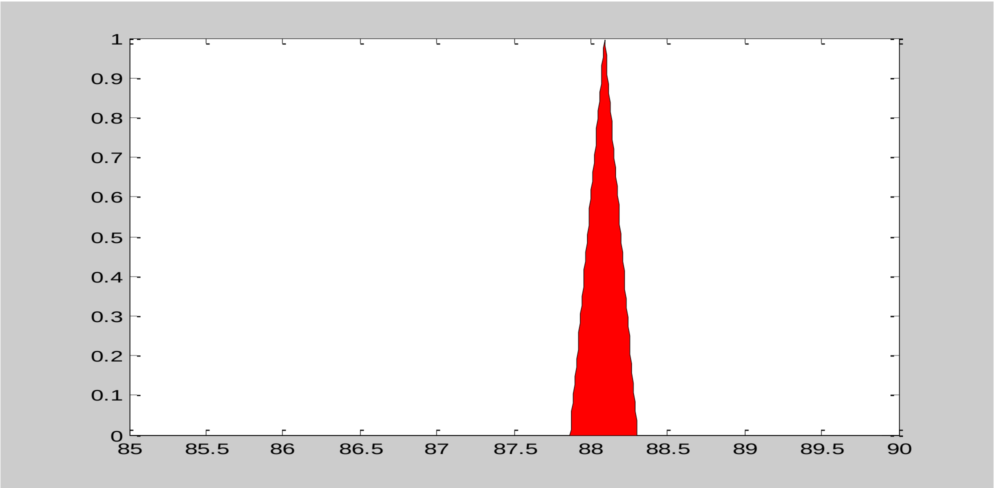


Figure 4.13: The OT fuzzy response at initial combination of factor levels.

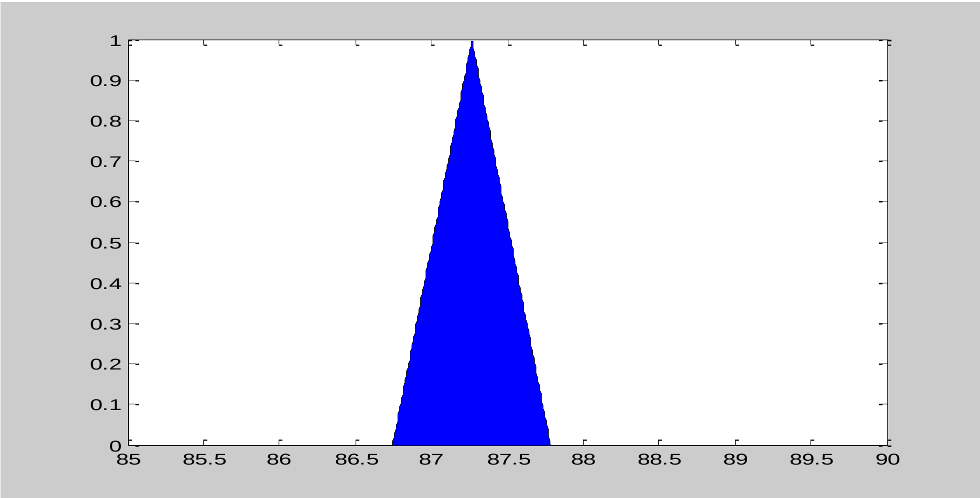


Figure 4.14: The OT fuzzy response at optimal combination of factor levels.

The anticipated improvements in three responses due to setting factor levels at the optimal combination expressed in Table 4.26 are calculated and listed in Table 4.27.

Table 4.27: Anticipated improvement for the sputtering process.

	Proposed approach		
	Initial fuzzy condition	Optimal fuzzy condition	Anticipated improvement
DR (dB)	(8.41, 8.70, 8.99)	(8.77, 12.26, 22.10)	(0.36 , 3.56, 13.11)
ER (dB)	(4.92, 5.49, 6.14)	(1.33, 3.12, 6.01)	(-3.59, -2.37, -0.13)
OT (dB)	(87.87, 88.09, 88.30)	(86.75, 87.27, 87.79)	(-1.12, -0.82, -0.51)
	Total anticipated improvement (dB)		**(-4.35, 0.37, 12.47)**

Case II: Steps 4-9, are conducted for trapezoidal membership function, and are described as follows:

The trapezoidal fuzzy regression $\tilde{\eta}_1$, $\tilde{\eta}_2$ and $\tilde{\eta}_3$ are obtained for the three responses and are respectively expressed as:

$\tilde{\eta}_1 =$ (10.2677, 10.3109, 10.3971, 10.4403) + (0.0796, 0.0797, 0.0800, 0.08022) X_1 + (0.3709, 0.3772, 0.3897, 0.396) X_2 + (0.0006, 0.0008, 0.0011, 0.0013) X_3 + (0.00001, 0.0008, 0.0023, 0.0031) X_4 + (0.0007, 0.00074, 0.00085, 0.0009) X_5 (4.28)

$\tilde{\eta}_2 =$ (-28.0398, -27.8814, -27.5646, -27.4062) + (0.1047, 0.1052, 0.1061, 0.1065) X_1 + (-0.0622, 0.0713, 0.3383, 0.4718) X_2 + (0.0104, 0.0105, 0.0108, 0.0109) X_3 + (0.0154, 0.0172, 0.0209, 0.0227) X_4 + (0.0112, 0.01121, 0.0113, 0.01132) X_5 (4.29)

and

$\tilde{\eta}_3 =$ (39.1999, 39.2030, 39.2093, 39.2124) + (-0.00239, -0.00238, -0.00235, -0.00233) X_1 + (-0.0603, -0.0567, -0.0496, -0.0461) X_2 + (-0.0048, -0.0047, -0.00465, -0.0046) X_3 + (0.001, 0.00102, 0.00107, 0.00109) X_4 + (0.00055, 0.00056, 0.000575, 0.00058) X_5 (4.30)

Based on the results of step 2, the optimal fuzzy factor levels $\tilde{X}^*_{fj}$ that optimize DR are as follows:

$\tilde{X}^*_{11}$ = (200, 200, 200,200), $\tilde{X}^*_{21}$ = (0.67, 0.67, 0.67, 0.67), $\tilde{X}^*_{31}$ = (60, 60, 60, 60),

$\tilde{X}^*_{41}$ = (100, 100, 100, 100) , $\tilde{X}^*_{51}$ = (100, 100, 100, 100)

Whereas, the optimal fuzzy factor levels $\tilde{X}^*_{fj}$ that optimize ER are as follows:

$\tilde{X}^*_{12}$ = (200, 200, 200, 200), $\tilde{X}^*_{22}$ = (0.67, 0.67, 0.67, 0.67), $\tilde{X}^*_{32}$ = (53.7868, 64.3934, 85.6066, 96.2132), $\tilde{X}^*_{42}$ = (100, 100, 100, 100), $\tilde{X}^*_{51}$ = (200, 200, 200, 200)

Finally, the optimal fuzzy factor levels $\tilde{X}^*_{fj}$ that optimize OT are as follows:

$\tilde{X}^*_{13}$ = (50, 50, 50, 50), $\tilde{X}^*_{23}$ = (0.13, 0.13, 0.13, 0.13), $\tilde{X}^*_{33}$ = (30, 30, 30, 30), $\tilde{X}^*_{43}$ = (100, 100, 100, 100), $\tilde{X}^*_{53}$ = (200, 200, 200, 200).

By substituting the optimal factor levels for the three responses in Eqs. (4.28) to (4.30), the values of the optimal triangular fuzzy responses are respectively expressed as:

$\tilde{\eta}^*_1$ = (26.5432, 26.7056, 27.0392, 27.2276)

$\tilde{\eta}^*_2$ = (-2.8021, -2.1555, -0.8434, -0.2114)

and

$\tilde{\eta}^*_3$ = (39.1386, 39.1496, 39.1679, 39.1769)

The values of pay-off matrix $\tilde{\eta}_j(x)$ for DR, ER, and OT are shown in Table 4.28.

Table 4.28: Pay-off matrix values for DR, ER, and OT.

	$\tilde{\eta}_1(X)$	$\tilde{\eta}_2(X)$	$\tilde{\eta}_3(X)$
$\tilde{X}_1^*$	(26.5432, 26.7056, 27.0392, 27.2276)	(-3.8575, -3.3226, -2.2499, -1.7361)	(38.5485, 38.5650, 38.5916, 38.6065)
$\tilde{X}_2^*$	(26.6095, 26.7831, 27.1524, 27.3647)	(-2.8021, -2.1555, -0.8434, -0.2114)	(38.4979, 38.5300, 38.6004, 38.6333)
$\tilde{X}_3^*$	(14.4549, 14.5969, 14.8808, 15.0318)	(-18.7209, -18.3351, -17.5416, -17.1629)	(39.1386, 39.1593, 39.1769)

The desirability function for each response is calculated using Eq. (3.6), where $\eta_{\min} = (12,12,12)$ and $\eta_{\max} = (30,30,30)$ are determine as an acceptable ranges for DR, while for ER, the acceptable ranges are $\eta_{\min} = (-24,-24,-24)$ and $\eta_{\max} = (0,0,0)$, finally, the acceptable ranges for OT are $\eta_{\min} = (38,38,38)$ and $\eta_{\max} = (40,40,40)$. Then, the pay-off matrix for desirability values is shown in Table 4.29.

Table 4.29: Pay-off matrix for desirability values for DR, ER, and OT.

	$\tilde{d}_{j1}(\tilde{\eta}_{j1}(X))$	$\tilde{d}_{j2}(\tilde{\eta}_{j2}(X))$	$\tilde{d}_{j3}(\tilde{\eta}_{j3}(X))$
$\tilde{X}_1^*$	(0.808, 0.8170, 0.8355, 0.8460)	(0.8393, 0.8616, 0.9063, 0.9277)	(0.2743, 0.2825, 0.2958, 0.3033)
$\tilde{X}_2^*$	(0.8116,0.8213, 0.8418, 0.8536)	(0.8832, 0.9102, 0.9649, 0.9912)	(0.2490, 0.2650, 0.3002, 0.3167)
$\tilde{X}_3^*$	(0.1364, 0.1443, 0.1600, 0.1684)	(0.2200, 0.2360, 0.2691, 0.2849)	(0.5693, 0.5748, 0.5840, 0.5885)

From Table 4.29, the following values are determined:

$$\tilde{U}_1 = \tilde{d}_{11} = (0.8080, 0.8170, 0.8355, 0.8460),\ \tilde{L}_1 = (0.1364, 0.1443, 0.1600, 0.1684)$$

$$\tilde{U}_2 = \tilde{d}_{22} = (0.8832, 0.9102, 0.9649, 0.9912),\ \tilde{L}_2 = (0.2200, 0.2360, 0.2691, 0.2849)$$

$$\tilde{U}_3 = \tilde{d}_{33} = (0.5693, 0.5748, 0.5840, 0.5885),\ \tilde{L}_3 = (0.2490, 0.2650, 0.3002, 0.3167)$$

The fuzzy deviation functions for DR, ER, and OT are expressed respectively as:

$$\tilde{D}_1 = 0.0864 + 0.00044\tilde{X}_1 + 0.0126\tilde{X}_2 + 0.0004\tilde{X}_3 + 0.0016\tilde{X}_4 + 0.0001\tilde{X}_5 \quad (4.31)$$

$$\tilde{D}_2 = 0.3168 + 0.0008\ \tilde{X}_1 + 0.267\ \tilde{X}_2 + 0.0002\ \tilde{X}_3 + 0.0036\ \tilde{X}_4 + 0.00004\tilde{X}_5 \quad (4.32)$$

$$\tilde{D}_3 = 0.0062 + 0.00004\tilde{X}_1 + 0.007\tilde{X}_2 + 0.0001\tilde{X}_3 + 0.00004\tilde{X}_4 + 0.00001\tilde{X}_5 \quad (4.33)$$

The pay-off matrix for deviation values is constituted in Table 4.30.

Table 4.30: Pay-off matrix for deviation values for DR, ER, and OT.

	$\tilde{D}_1(X)$	$\tilde{D}_2(X)$	$\tilde{D}_3(X)$
$\tilde{X}_1^*$	(0.3768, 0.3768, 0.3768, 0.768)	(1.0317, 1.0317, 1.0317, 1.0317)	(0.0300, 0.0300, 0.0300, 0.0300)
$\tilde{X}_2^*$	(0.3844, 0.886, 0.3971, 0.4013)	(1.0344, 1.0366, 1.0408, 1.0429)	(0.0303, 0.0313, 0.0335, 0.0350)
$\tilde{X}_3^*$	(0.3020, 0.3020, 0.3020, 0.3020)	(0.7655, 0.7655, 0.7655, 0.7655)	(0.0181, 0.0181, 0.0181, 0.0181)

From Table 4.30, the following values are expressed as:

$$\tilde{P}_1 = (0.3768, 0.3768, 0.3768, 0.3768),\ \tilde{Q}_1 = (0.3844, 0.3886, 0.3971, 0.4013)$$

$$\tilde{P}_2 = (1.0344, 1.0366, 1.0408, 1.0429),\ \tilde{Q}_2 = (1.0344, 1.0366, 1.0408, 1.0429)$$

$$\tilde{P}_3 = (0.0181, 0.0181, 0.0181),\ \tilde{Q}_3 = (0.0303, 0.0313, 0.0335, 0.0345)$$

The final model is constructed as:

$$\begin{aligned} &Max\ \{\tilde{d}_1(X), \tilde{d}_2(X), \tilde{d}_3(X)\} \\ &Min\ \{D_1(X), D_2(X), D_3(X)\} \\ &s.t \\ &X = \{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5\} \in [\text{Factor Levels}] \end{aligned}$$

Since it is desirable to maximize these two functions, the objective function is formulated as follows:

$$
\begin{aligned}
&Max\ \{\tilde{S}_1(X),\tilde{S}_2(X),\tilde{S}_3(X)\}\\
&Max\ \{\tilde{T}_1(X),\tilde{T}_2(X),\tilde{T}_3(X)\}\\
&s.t\\
&X=\{\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{x}_4,\tilde{x}_5\}\in\text{[Factor Levels]}
\end{aligned}
$$

By applying Zimmerman Max-Min operator, the two objectives is converted to single objective, this maximizes the minimum degree of satisfaction from the two objectives. Let $Min\ \tilde{S}_j(X)=\tilde{S}=(S^l,S^b,S^c,S^u)$, $Min\ \tilde{T}_j(X)=\tilde{T}=(T^l,T^b,T^c,T^u)$. Then, the problem is formulated as follows:

$$
\begin{aligned}
&Max\quad \tilde{S}(X)\\
&Max\quad \tilde{T}(X)\\
&s.t\\
&\tilde{d}_j(X)-\tilde{S}(\tilde{U}_j-\tilde{L}_j)\geq\tilde{L}_j \qquad ,j=1,2,3.\\
&\tilde{D}_j(X)+\tilde{T}(\tilde{Q}_j-\tilde{P}_j)\leq\tilde{Q}_j \qquad ,j=1,2,3.\\
&X=\{\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{x}_4,\tilde{x}_5\}\in\text{[Factor Levels]}
\end{aligned}
$$

Finally, the final model using Eq. (3.13) is categorized to four types: model l, model b, model c, and model u, where models l and u are solved in triangular membership function, whereas the remaining models as shown below:

Model b **:**

$$
\begin{aligned}
&Max\quad 0.5\times S^b+0.5\times T^b\\
&s.t\\
&-0.09384+0.00443x_1^b+0.021x_2^b+4.44\times10^{-5}x_3^b+4.44\times10^{-5}x_4^b+4.11\times10^{-5}x_5^b-0.6727S^b\geq0.1443\\
&-0.16173+0.00438x_1^b-0.00297x_2^b+4.375\times10^{-4}x_3^b+7.17\times10^{-4}x_4^b+4.67\times10^{-4}x_5^b-0.6742S^b\geq0.236\\
&0.6015-0.00119x_1^b-0.0284x_2^b-0.00235x_3^b+0.00051x_4^b+2.8\times10^{-4}x_5^b-0.3098S^b\geq0.2650\\
&0.0863+0.00044x_1^b+0.126x_2^b+0.0004x_3^b+0.0016x_4^b+0.0001x_5^b+0.0118T^b\leq0.3886\\
&0.3168+0.0008x_1^b+0.267x_2^b+0.0002x_3^b+0.0036x_4^b+0.00004x_5^b+0T^b\leq1.0366\\
&0.0062+0.00004x_1^b+0.007x_2^b+0.0001x_3^b+0.00004x_4^b+0.00001x_5^b+0.0132T^b\leq0.0313\\
&w_1+w_2=1\\
&0\leq S^b,T^b\leq1\\
&X=\{x_1^b,x_2^b,x_3^b,x_4^b,x_5^b\}\in\text{[Factor Levels]}
\end{aligned}
$$

Model c :

$$
\begin{aligned}
& Max \quad 0.5\times S^c + 0.5\times T^c \\
& s.t \\
& \text{-}0.0867 + 0.0045x_1^c + 0.022x_2^c + 7.22\times10^{-5}x_3^c + 1.72\times10^{-4}x_4^c + 5\times10^{-5}x_5^c - 0.6776S^c \geq 0.1684 \\
& \text{-}0.1419 + 0.0044x_1^c + 0.0197x_2^c + 4.5\times10^{-4}x_3^c + 9.46\times10^{-4}x_4^c + 4.71\times10^{-4}x_5^c - 0.7063S^c \geq 0.2849 \\
& 0.6062 - 0.00117x_1^c - 0.0231x_2^c - 0.0023x_3^c + 5.45\times10^{-4}x_4^c + 2.9\times10^{-4}x_5^c - 0.2718S^c \geq 0.3167 \\
& 0.0864 + 0.00044x_1^c + 0.126x_2^c + 0.0004x_3^c + 0.0016x_4^c + 0.0001x_5^c + 0.0245T^c \leq 0.4013 \\
& 0.3168 + 0.0008x_1^c + 0.267x_2^c + 0.0002x_3^c + 0.0036x_4^c + 0.00004x_5^c + 0T^c \leq 1.0429 \\
& 0.0062 + 0.00004x_1^c + 0.0071x_2^c + 0.0001x_3^c + 0.00004x_4^c + 0.00001x_5^c + 0.0164T^c \leq 0.0345 \\
& w_1 + w_2 = 1 \\
& 0 \leq S^c, T^c \leq 1 \\
& X = \left\{x_1^c, x_2^c, x_3^c, x_4^c, x_5^c\right\} \in [\text{Factor Levels}]
\end{aligned}
$$

The four models on $w_1 = w_1 = 0.5$, for desirability and robustness are solved, then the optimal factor levels of models l, m, and u are determined as shown in Table 4.31 as follows:

Table 4.31: The optimal factor levels for the sputtering process.

	Model *l*	Model *b*	Model *c*	Model *u*
x_1^*	139.8411	140.7334	142.5244	200
x_2^*	0.13	0.13	0.13	0.13
x_3^*	30	30	30	30
x_4^*	25	25	25	25
x_5^*	0	200	200	200

By substituting the values of factor levels from models *l*, *b*, *c*, and *u*, in Eqs. (4.28) to (4.30), the results are respectively expressed as:

$\tilde{\eta}_1^*$ = (21.4655, 21.7984, 22.1102, 26.8258)

$\tilde{\eta}_2^*$ = (-12.7095, -10.0820, -9.2923, -2.8904)

$\tilde{\eta}_3^*$ = (38.7388, 38.7457, 38.8572, 38.8702)

By converting the $\tilde{\eta}_1^*$ to $\tilde{y}_1^*$, $\tilde{\eta}_2^*$ to $\tilde{y}_2^*$, and $\tilde{\eta}_3^*$ to $\tilde{y}_3^*$ using Eqs. (4.16) to (4.18), the optimal fuzzy responses are respectively obtained as:

$\tilde{y}_1^*$ = (11.8379, 12.2580, 12.7500, 21.9427)

$\tilde{y}_2^*$ = (1.3948, 2.9148, 3.1923, 4.3199)

$\tilde{y}_3^*$ = (86.4848, 86.5536, 87.6718, 87.8031)

The factor levels for initial condition were set at the combination $X1_{(2)}X2_{(2)}X3_{(1)}X4_{(3)}X5_{(1)}$. By substituting the factor levels for this combination in Eqs. (4.28) to (4.30), respectively, then converting the results to $\tilde{y}_1$, $\tilde{y}_2$, and $\tilde{y}_3$ using Eqs. (4.16) to (4.18), respectively, the obtained fuzzy responses at initial combination of factor levels are written as:

$\tilde{y}_1$ = (8.4180, 8.5483, 8.8320, 8.9915)

$\tilde{y}_2$ = (4.9222, 5.1963, 5.8067, 6.1373)

$\tilde{y}_3$ = (87.8668, 87.9832, 88.1922, 88.3029)

Plotting the DR fuzzy response at initial combination of factor levels, $\tilde{y}_1$, and at optimal combination of factor levels, $\tilde{y}_1^*$, is represented in Figures 4.15 and 4.16, respectively. Similarly, plotting ER fuzzy response $\tilde{y}_2 / \tilde{y}_2^*$ at initial/ optimal combination of factor levels are represented in Figures 4.17, 4.18, respectively, finally plotting OT fuzzy response $\tilde{y}_3 / \tilde{y}_3^*$ at initial/ optimal combination of factor levels are shown in Figures 4.19, and 4.20, respectively.

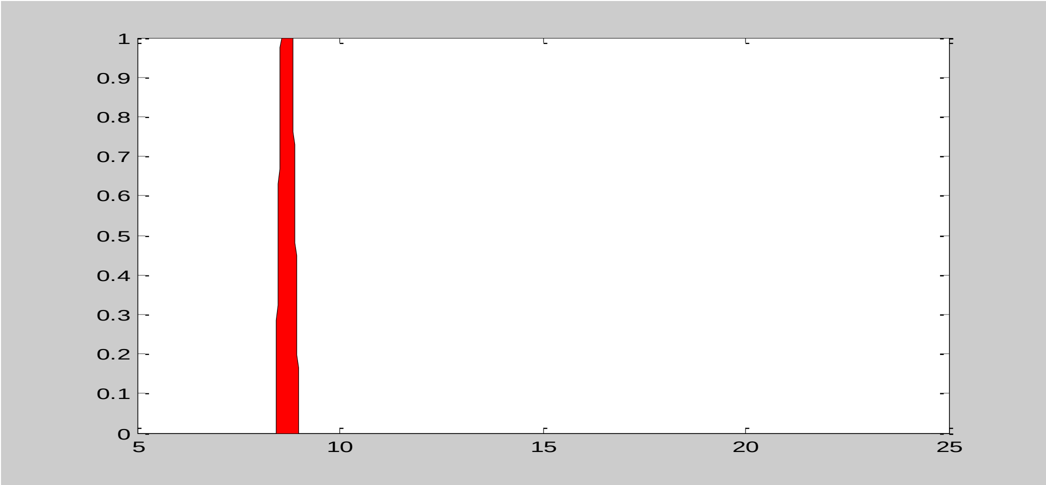


Figure 4.15: The DR trapezoidal fuzzy response at initial combination of factor levels.

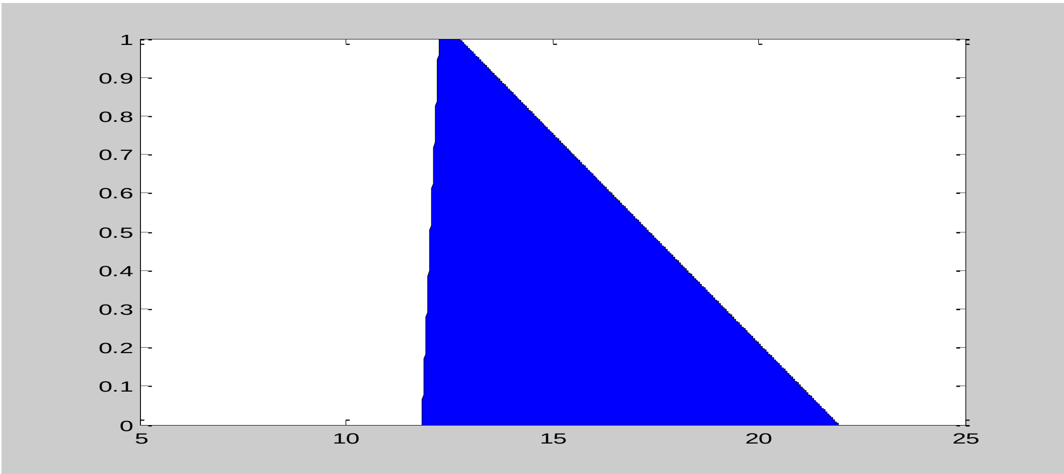


Figure 4.16: The DR trapezoidal fuzzy response at optimal combination of factor levels.

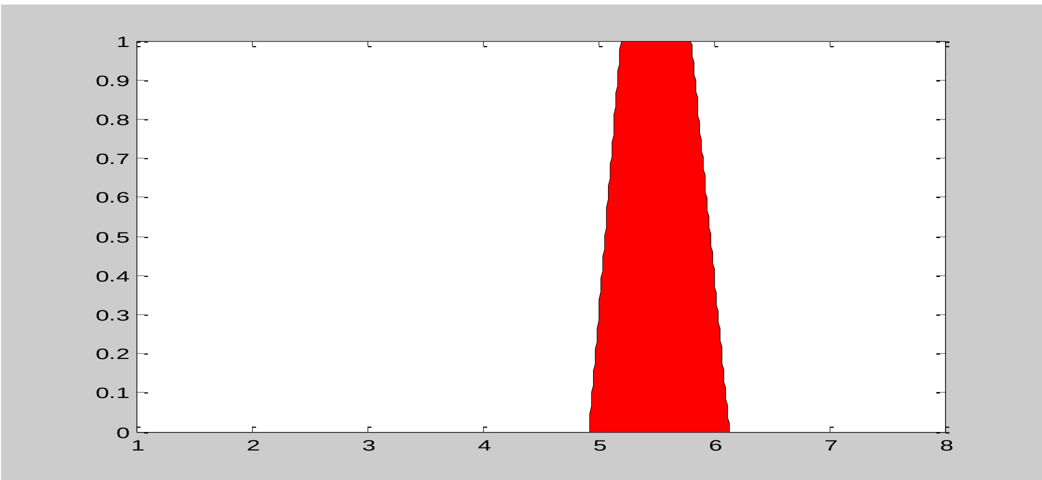


Figure 4.17: The ER trapezoidal fuzzy response at initial combination of factor levels.

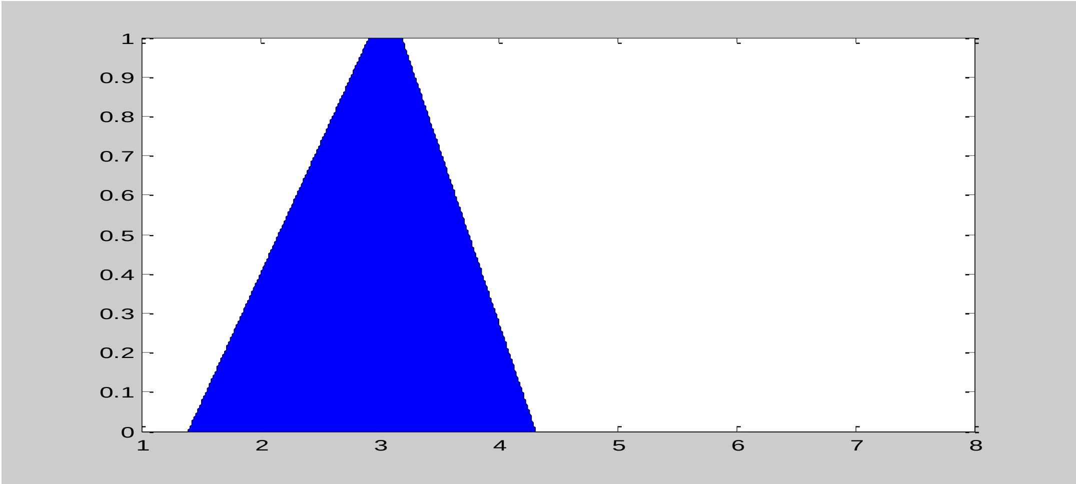


Figure 4.18: The ER trapezoidal fuzzy response at optimal combination of factor levels.

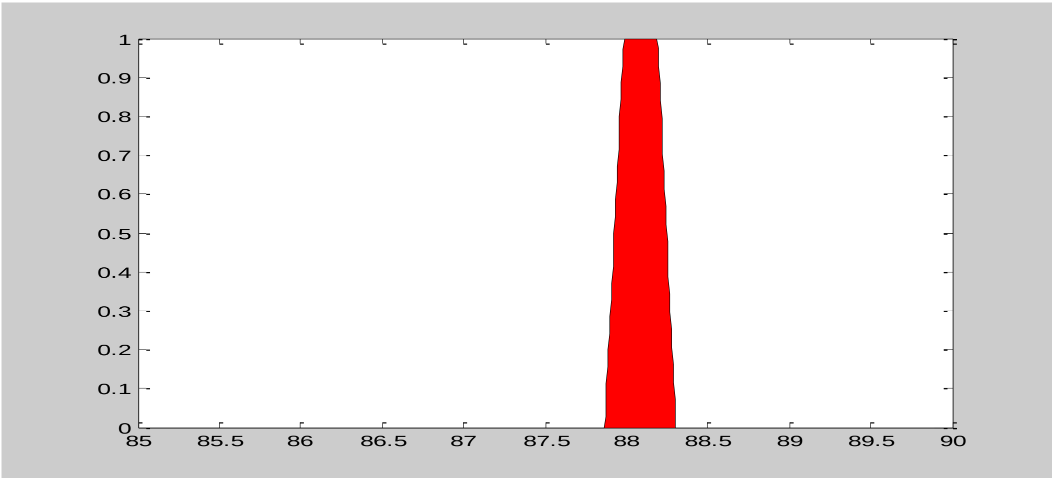


Figure 4.19: The OT trapezoidal fuzzy response at initial combination of factor levels.

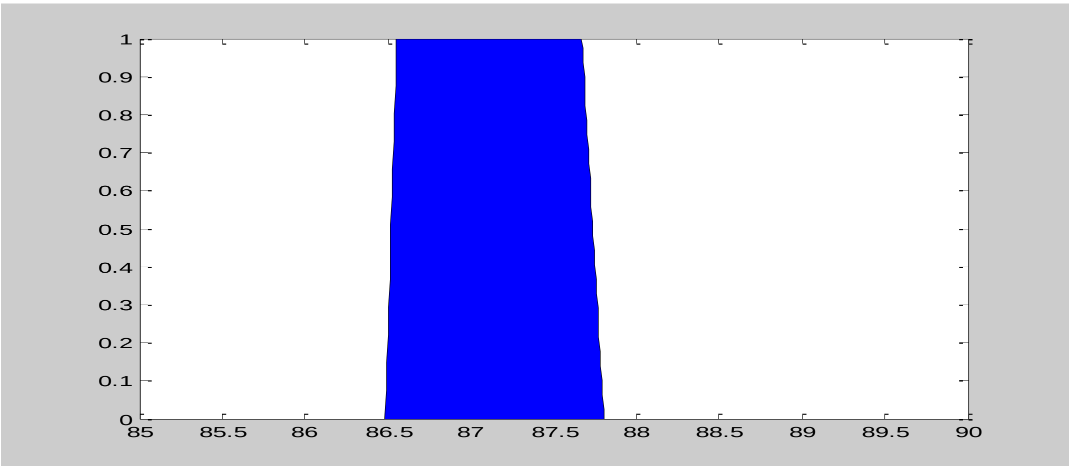


Figure 4.20: The OT trapezoidal fuzzy response at optimal combination of factor levels.

The anticipated improvements in three responses due to setting factor levels at the optimal combination expressed in Table 4.31 are calculated and listed in Table 4.32.

Table 4.32: Anticipated improvement for the sputtering process.

		Proposed approach	
	Initial fuzzy condition	Optimal fuzzy condition	Anticipated improvement
DR (dB)	(8.42, 8.55, 8.83, 8.99)	(11.84, 12.26, 12.75, 21.94)	(3.42 , 3.71, 3.92, 12.95)
ER (dB)	(4.92, 5.20, 5.81, 6.14)	(1.39, 2.91, 3.19, 4.32)	(-3.53, -2.29, -2.62, -1.82)
OT (dB)	(87.87, 87.98, 88.19, 88.3)	(86.48, 86.55, 87.67, 87.80)	(-1.39, -1.43, -0.52, -0.5)
	Total anticipated improvement (dB)		**(-1.5, 0.01, 0.78, 10.63)**

Comparing between triangular and trapezoidal membership functions, the following results are obtained:

(i). The proposed approach with each of the two membership functions provides a larger satisfactory region than the initial condition.

(ii). The triangular membership function increases the satisfactory region, and represents the ideal (mean) solution as one value; however, trapezoidal membership function increases the flexibility of the fuzzy models, and represents the ideal (mean) solution as ranges. Table 4.31 the ideal solution for three responses DR, ER, and OT.

Table 4.33: The ideal solution for DR, ER, and OT responses.

Responses	**Triangular Ideal Solution**	**Trapezoidal Ideal Solution**
DR	(12.26)	[12.26-12.75]
ER	(3.11)	[2.91-3.19]
OT	(87.2680)	[86.55- 87.67]

(iii). Both membership functions deviation and fuzziness, and all inherent variability in the collected data have been minimized and optimized, by considering each replicate, in contrast with the traditional regression approaches.

4.3. Optimization of BOD conditions to improve water and wastewater quality of biodegradation process

The quality of, conventional and non-conventional, water is a key issue in the proper management of water resources. One of the parameters that are determining water quality is the Biochemical Oxygen Demand (BOD), which is a measure of the aerobically biodegradable pollutants "organic matter". This test is applied to test the quality of surface water and wastewaters, and thus is included as a main parameter in the standards of water quality and the standards of reclaimed wastewaters for reuse. In addition to the BOD test is the basis to select, design, operate and evaluate the efficiency of different treatment technologies, which treat water and wastewaters. Four process factors are investigated concurrently using L_{27} array, including: (X_1) dilution "substrate concentrations", (X_2) seed "microorganisms", (X_3) temperature, and (X_4) pH, on two responses: real biodegradability (RB) and microorganism (MO) of biodegradation process, where (y_1, LTB) and (y_2, STB), respectively, the experimental data are displayed in Table 4.34, and factors and their levels for experiment can be seen in Table 4.35.

Table 4.34: Orthogonal array and the experimental data for biodegradation process.

Exp. *i*	**Factor levels**				**RB**		**MO**	
	X_1	X_2	X_3	X_4	y_{i11}	y_{i12}	y_{i21}	y_{i22}
1	1	1	1	1	0.76	0.80	0.18	0.14
2	1	1	2	2	0.80	0.78	0.11	0.12
3	1	1	3	3	0.31	0.29	0.38	0.39
4	1	2	1	2	0.72	0.71	0.19	0.19
5	1	2	2	3	0.81	0.84	0.11	0.13
6	1	2	3	1	0.85	0.85	0.12	0.13
7	1	3	1	3	0.82	0.77	0.12	0.10
8	1	3	2	1	0.86	0.85	0.09	0.07
9	1	3	3	2	0.80	0.81	0.14	0.15
10	2	1	1	2	0.27	0.14	0.28	0.27
11	2	1	2	3	0.73	0.75	0.24	0.25
12	2	1	3	1	0.45	0.46	0.21	0.20
13	2	2	1	3	0.67	0.71	0.32	0.24
14	2	2	2	1	0.83	0.82	0.10	0.12
15	2	2	3	2	0.56	0.53	0.26	0.25
16	2	3	1	1	0.82	0.83	0.13	0.13
17	2	3	2	2	0.81	0.86	0.09	0.06
18	2	3	3	3	0.92	0.92	0.06	0.06
19	3	1	1	3	0.80	0.80	0.14	0.14
20	3	1	2	1	0.82	0.85	0.12	0.12
21	3	1	3	2	0.84	0.85	0.13	0.10
22	3	2	1	1	0.81	0.82	0.16	0.13
23	3	2	2	2	0.85	0.82	0.23	0.20
24	3	2	3	3	0.85	0.86	0.11	0.12
25	3	3	1	2	0.83	0.84	0.14	0.13
26	3	3	2	3	0.82	0.85	0.10	0.10
27	3	3	3	1	0.89	0.89	0.10	0.10

Table 4.35: Factors and their levels for experiment for biodegradation process.

Factor	**Process variables**	**Level 1**	**Level 2**	**Level 3**
X_1	Substrate concentrations (mg Liter^{-1})	400	600	800
X_2	Microorganisms (%)	10	20	30
X_3	Temperature (C^{o})	20	28.5	37
X_4	pH	6.5	7.5	8.5

Let η_{i1r} and η_{i2r} denote the S/N ratio for RB and MO responses at experiment i (i =1,…, 27) with replicate r (r =1, 2), respectively. Then, the η_{i1r} and η_{i2r} are calculated respectively for each replicate as follows:

$$\eta_{i1r} = -10\, log_{10}(1/\, y_{i1r}^{2}) \qquad i = 1,...,27. \quad r = 1,2. \qquad \text{for RB} \qquad (4.34)$$

$$\eta_{i2r} = -10\log_{10}(y_{i2r}^{2}) \qquad i = 1,...,27. \quad r = 1,2. \qquad \text{for MO} \qquad (4.35)$$

The obtained η_{i1r} and η_{i2r} for each replicate are summarized in Table 4.36.

Table 4.36: The experimental results for each replicate for biodegradation process.

Exp. i	η_{i11}	η_{i12}	η_{i21}	η_{i22}
1	-2.384	-1.938	14.895	17.077
2	-1.938	-2.158	19.172	18.416
3	-10.17	-10.750	8.4043	8.1787
4	-2.853	-2.975	14.425	14.425
5	-1.83	-1.514	19.172	17.721
6	-1.412	-1.412	18.416	17.721
7	-1.724	-2.270	18.416	20.000
8	-1.310	-1.412	20.915	23.098
9	-1.938	-1.830	17.077	16.478
10	-11.37	-17.077	11.057	11.373
11	-2.734	-2.499	12.396	12.041
12	-6.936	-6.745	13.556	13.979
13	-3.479	-2.975	9.897	12.396
14	-1.618	-1.724	20.000	18.416
15	-5.036	-5.514	11.701	12.041
16	-1.724	-1.618	17.721	17.721
17	-1.830	-1.310	20.915	24.437
18	-0.724	-0.724	24.437	24.437
19	-1.938	-1.938	17.077	17.077
20	-1.724	-1.412	18.416	18.416
21	-1.514	-1.412	17.721	20.000
22	-1.83	-1.724	15.918	17.721
23	-1.412	-1.724	12.765	13.979
24	-1.412	-1.310	19.172	18.416
25	-1.618	-1.514	17.077	17.721
26	-1.724	-1.412	20.000	20.000
27	-1.012	-1.012	20.000	20.000

Table 4.37: The optimal factor levels for the two replicates of RB.

Factor (f)	Replicate r=1			Replicate r=2		
	Level 1	Level 2	Level 3	Level 1	Level 2	Level 3
X_1	-2.840	-3.939	-1.576	-2.918	-4.465	-1.495
X_2	-4.524	-2.320	-1.512	-5.103	-2.319	-1.456
X_3	-3.214	-1.791	-3.351	-3.781	-1.685	-3.412
X_4	-2.111	-3.946	-2.822	-2.111	-3.946	-2.822

Table 4.38: The optimal factor levels for the two replicates of MO.

Factor (f)	Replicate r=1			Replicate r=2		
	Level 1	Level 2	Level 3	Level 1	Level 2	Level 3
X_1	16.77	15.74	17.57	17.01	16.32	18.15
X_2	14.74	15.72	19.62	15.17	15.87	20.43
X_3	15.16	18.19	16.72	16.17	18.50	16.81
X_4	17.76	15.77	16.55	18.24	16.54	16.70

The multiple linear regression equations for η_{11} and η_{12} are respectively written as:

$$\eta_{11} = -5.052 + 0.00316X_1 + 0.1506X_2 - 0.00807X_3 - 0.3215X_4 \quad (4.36.a)$$

and

$$\eta_{12} = -6.693 + 0.00356X_1 + 0.18238X_2 + 0.02169X_3 - 0.3555X_4 \quad (4.36.b)$$

Whereas, the multiple linear regression equations for η_{21} and η_{22} are respectively represented by:

$$\eta_{21} = 12.529 + 0.00202X_1 + 0.2437X_2 + 0.09151X_3 - 0.6036X_4 \quad (4.37.a)$$

and

$$\eta_{22} = 14.913 + 0.00284X_1 + 0.26296X_2 + 0.03752X_3 - 0.7713X_4 \quad (4.37.b)$$

Case I: Steps 4-9, are conducted for triangular membership function, and are described as follow:

The triangular fuzzy regression $\tilde{\eta}_1$ and $\tilde{\eta}_2$ are obtained for the two responses, respectively, and expressed as:

$\tilde{\eta}_1 =$ (-7.0329, -5.8725, -4.7121) + (0.0031, 0.0034, 0.0036) X_1 + (0.1440, 0.1665, 0.1890) X_2 + (-0.0142, 0.0068, 0.0279) X_3 + (-0.3623, -0.3385, -0.3145) X_4 (4.38)

and

$\tilde{\eta}_2 =$ (12.0353, 13.721, 15.4067) + (0.0019, 0.0024, 0.003) X_1 + (0.2397, 0.2533, 0.2670) X_2 + (0.0263, 0.0645, 0.1027) X_3 + (-0.806, -0.6875, -0.5689) X_4 (4.39)

Based on the results of step 2, the optimal fuzzy factor levels $\tilde{X}^*_{fj}$ that optimize RB response are as follows:

$\tilde{X}^*_{11}$ = (800, 800, 800), $\tilde{X}^*_{21}$ = (30, 30, 30), $\tilde{X}^*_{31}$ = (28.5, 28.5, 28.5), $\tilde{X}^*_{41}$ = (6.5, 6.5, 6.5).

Whereas the optimal fuzzy factor levels $\tilde{X}^*_{fj}$ that optimize MO response are as follows:

$\tilde{X}^*_{12}$ = (800, 800, 800), $\tilde{X}^*_{22}$ = (30, 30, 30), $\tilde{X}^*_{32}$ = (28.5, 28.5, 28.5), $\tilde{X}^*_{42}$ = (6.5, 6.5, 6.5).

By substituting the optimal factor levels for the two responses in Eqs. (4.38) and (4.39), the values of the optimal triangular fuzzy responses are respectively expressed as:

$\tilde{\eta}^*_1$ = (-2.9939, -0.1640, 2.5888).

and

$\tilde{\eta}^*_2$ = (16.2569, 20.6095, 25.0428).

The values of pay-off matrix $\tilde{\eta}_j(x)$ for both responses are shown in Table 4.39.

Table 4.39: Pay-off matrix values for RB and MO.

	$\tilde{\eta}_1(X)$	$\tilde{\eta}_2(X)$
$\tilde{X}^*_1$	(-2.9939, -0.1640, 2.5888)	(16.2569, 20.6095, 25.0428)
$\tilde{X}^*_2$	(-2.9939, -0.1640, 2.5888)	(16.2569, 20.6095, 25.0428)

The desirability function for each response is calculated using Eq. (3.6), where $\eta_{\min}=(-12)$, $\eta_{\max}=(0)$ for RB and $\eta_{\min}=(8)$, $\eta_{\max}=(25)$ for MO are determined as an acceptable ranges. Then, the pay-off matrix for desirability values is shown in Table 4.40.

Table 4.40: Pay-off matrix for desirability values for RB and MO.

	$\tilde{d}_1(\tilde{\eta}_1(X))$	$\tilde{d}_2(\tilde{\eta}_2(X))$
$\tilde{X}^*_1$	(0.7505, 0.9986, 1)	(0.4857, 0.7417, 1)
$\tilde{X}^*_2$	(0.7505, 0.9986, 1)	(0.4857, 0.7417, 1)

The following values from Table 4.36 are determined as:

$\tilde{U}_1=(0.7505, 0.9986, 1)$, $\tilde{L}_1=(0.7505, 0.9986, 1)$ $\tilde{U}_2=(0.4857, 0.7417, 1)$,

$\tilde{L}_2=(0.4857, 0.7417, 1)$

The fuzzy deviation functions for RB and MO are then expressed respectively as:

$$\tilde{D}_1(X)=1.1604+2\times10^{-4}\tilde{X}_1+0.0226\tilde{X}_2+0.0212\tilde{X}_3+0.024\tilde{X}_4. \quad (4.40)$$

and

$$\tilde{D}_2(X)=1.6856+6\times10^{-4}\tilde{X}_1+0.0136\tilde{X}_2+0.0382\tilde{X}_3+0.1186\tilde{X}_4. \quad (4.41)$$

The pay-off matrix for deviation values is constituted as shown in Table 4.41.

Table 4.41: Pay-off matrix for deviation values for RB and MO.

	$\tilde{D}_1(X)$	$\tilde{D}_2(X)$
$\tilde{X}^*_1$	(2.7586, 2.7586, 2.7586)	(4.4332, 4.4332, 4.4332)
$\tilde{X}^*_2$	(2.7586, 2.7586, 2.7586)	(4.4332, 4.4332, 4.4332)

From Table 4.41, the following values are expressed as:

$\tilde{P}_1 = (2.7586,\ 2.7586,\ 2.7586),\ \tilde{Q}_1 = (2.7586,\ 2.7586,\ 2.7586),$

$\tilde{P}_2 = (4.4332,\ 4.4332,\ 4.4332),\ \tilde{Q}_2 = (4.4332,\ 4.4332,\ 4.4332).$

The final model is constructed as:

$$\begin{aligned} &Max\ \left\{\tilde{d}_1(X),\tilde{d}_2(X)\right\} \\ &Min\left\{D_1(X),D_2(X)\right\} \\ &s.t \\ &X = \left\{\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{x}_4\right\} \in [\text{Factor Levels}] \end{aligned}$$

Since it is desirable to maximize these two functions, the objective function is formulated as follows:

$$\begin{aligned} &Max\ \left\{\tilde{S}_1(X),\tilde{S}_2(X)\right\} \\ &Max\ \left\{\tilde{T}_1(X),\tilde{T}_2(X)\right\} \\ &s.t \\ &X = \left\{\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{x}_4\right\} \in [\text{Factor Levels}] \end{aligned}$$

By applying Zimmerman Max-Min operator, the two objectives is converted to single objective, this maximizes the minimum degree of satisfaction from the two objectives. Let $Min\quad \tilde{S}_j(x) = \tilde{S} = (S^l, S^m, S^u),\ Min\quad \tilde{T}_j(x) = \tilde{T} = (T^l, T^m, T^u)$. Then, the problem is formulated as follows:

$$\begin{aligned} &Max\quad \tilde{S}(X) \\ &Max\quad \tilde{T}(X) \\ &s.t \\ &\tilde{d}_j(X) - \tilde{S}(\tilde{U}_j - \tilde{L}_j) \geq \tilde{L}_j \qquad , j = 1,2. \\ &\tilde{D}_j(X) + \tilde{T}(\tilde{Q}_j - \tilde{P}_j) \leq \tilde{Q}_j \qquad , j = 1,2. \\ &X = \left\{\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,\tilde{x}_4\right\} \in [\text{Factor Levels}] \end{aligned}$$

Finally, the final model using Eq. (3.13) is categorized to three types: model l, model m, model u as shown below:

Model l **:**

$$
\begin{aligned}
&Max \quad 0.5\times S^{l}+0.5\times T^{l}\\
&s.t\\
&0.41393+2.565\times10^{-4}x_1^{l}+0.02x_2^{l}-1.1862\times10^{-3}x_3^{l}-0.03021x_4^{l}-0S^{l}\geq 0.7505\\
&0.2374+1.08824\times10^{-4}x_1^{l}+0.0141x_2^{l}+1.5494\times10^{-3}x_3^{l}-0.0474x_4^{l}-0\,S^{l}\geq 0.4857\\
&1.1604+2\times10^{-4}x_1^{l}+0.0226x_2^{l}+0.0212x_3^{l}+0.024x_4^{l}+0\,T^{l}\leq 2.7586\\
&1.6857+6\times10^{-4}x_1^{l}+0.0136x_2^{l}+0.0382x_3^{l}+0.1186x_4^{l}+0\,T^{l}\leq 4.4332\\
&w_1+w_2=1\\
&0\leq S^{l},T^{l}\leq 1\\
&X=\left\{x_1^{l},x_2^{l},x_3^{l},x_4^{l}\right\}\in[\text{Factor Levels}]
\end{aligned}
$$

Model m :

$$
\begin{aligned}
&Max \quad 0.5\times S^{m}+0.5\times T^{m}\\
&s.t\\
&0.51063+2.7992\times10^{-4}x_1^{m}+0.01387x_2^{m}+5.675\times10^{-4}x_3^{m}-0.02821x_4^{m}-0S^{m}\geq 0.9986\\
&0.33653+1.4294\times10^{-4}x_1^{m}+0.0149x_2^{m}+3.7953\times10^{-3}x_3^{m}-0.04044x_4^{m}-0\,S^{m}\geq 0.7417\\
&1.1604+2\times10^{-4}x_1^{m}+0.0226x_2^{m}+0.0212x_3^{m}+0.024x_4^{m}+0\,T^{m}\leq 2.7586\\
&1.6857+6\times10^{-4}x_1^{m}+0.0136x_2^{m}+0.0382x_3^{m}+0.1186x_4^{m}+0\,T^{m}\leq 4.4332\\
&w_1+w_2=1\\
&0\leq S^{m},T^{m}\leq 1\\
&X=\left\{x_1^{m},x_2^{m},x_3^{m},x_4^{m}\right\}\in[\text{Factor Levels}]
\end{aligned}
$$

Model u :

$$
\begin{aligned}
&Max \quad 0.5\times S^{u}+0.5\times T^{u}\\
&s.t\\
&0.6073+3.0325\times10^{-4}x_1^{u}+0.01575x_2^{u}+2.321\times10^{-3}x_3^{u}-0.02621x_4^{u}-0S^{u}\geq 1\\
&0.4357+1.7706\times10^{-4}x_1^{u}+0.015703x_2^{u}+6.0406\times10^{-3}x_3^{u}-0.03346x_4^{u}-0\,S^{u}\geq 1\\
&1.1604+2\times10^{-4}x_1^{u}+0.0226x_2^{u}+0.0212x_3^{u}+0.024x_4^{u}+0\,T^{u}\leq 2.7586\\
&1.6857+6\times10^{-4}x_1^{u}+0.0136x_2^{u}+0.0382x_3^{u}+0.1186x_4^{u}+0\,T^{u}\leq 4.4332\\
&w_1+w_2=1\\
&0\leq S^{u},T^{u}\leq 1\\
&X=\left\{x_1^{u},x_2^{u},x_3^{u},x_4^{u}\right\}\in[\text{Factor Levels}]
\end{aligned}
$$

The three models on $w_1 = w_1 = 0.5,$ for desirability and robustness are solved, then the optimal factor levels of models l, m, and u are determined as shown in Table 4.42.

Table 4.42: The optimal factor levels for biodegradation process.

	Model *l*	Model *m*	Model *u*
X_1^*	780.7478	800	800
X_2^*	30	30	30
X_3^*	28.5	28.5	28.6816
X_4^*	6.5	6.5	6.5

Substituting the values the of factor levels from model *l*, model *m*, and model *u*, respectively in Eqs. (4.38) and (4.39), the results are respectively expressed as:

$\tilde{\eta}_1^*$ = (-3.0522, -0.1640, 2.5939)

and

$\tilde{\eta}_2^*$ = (16.2203, 20.6095, 25.0645)

By converting the $\tilde{\eta}_1^*$ to $\tilde{y}_1^*$ using Eq. (4.34) and converting $\tilde{\eta}_2^*$ to $\tilde{y}_2^*$ using Eq. (4.35), the optimal fuzzy responses are respectively obtained:

$\tilde{y}_1^*$ = (0.7037, 0.9813, 1.3480)

and

$\tilde{y}_2^*$ = (0.0558, 0.0932, 0.1545)

Plotting the RB triangular fuzzy response at optimal combination of factor levels $\tilde{y}_1^*$, is represented in Figure 4.21. Similarly, plotting the MO triangular fuzzy response at optimal combination of factor levels $\tilde{y}_2^*$, is represented in Figure 4.22.

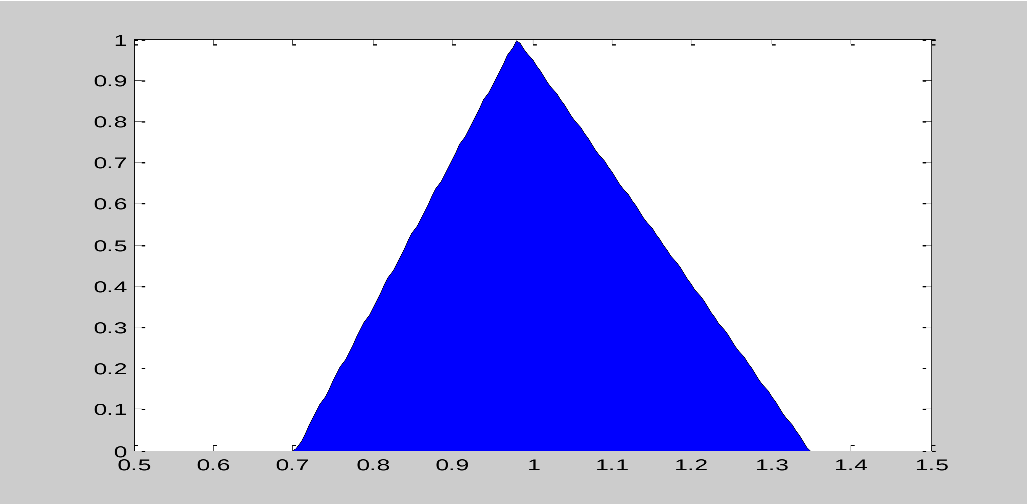


Figure 4.21: The RB fuzzy response at optimal combination of factor levels.

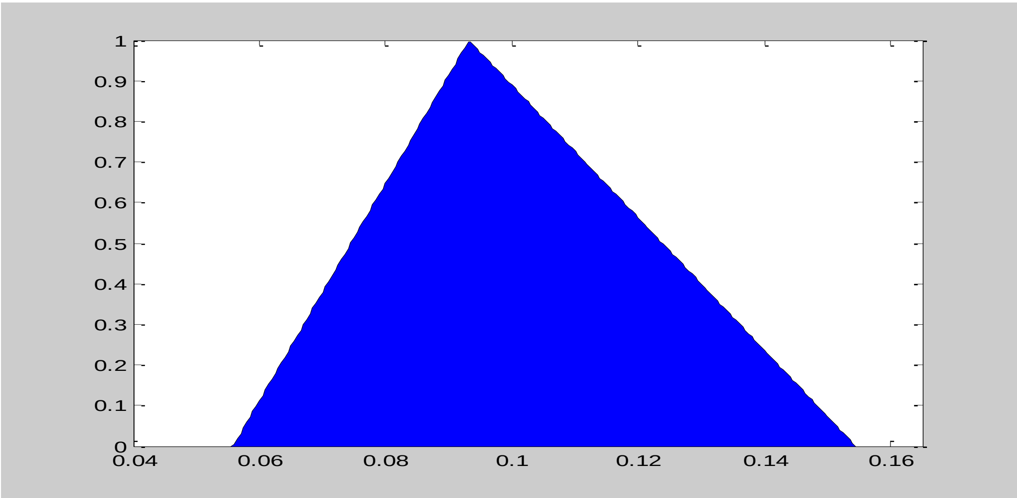


Figure 4.22: The MO fuzzy response at optimal combination of factor levels.

Case II: Steps 4-9, are conducted for trapezoidal membership function, and are described as follows:

The trapezoidal fuzzy regression $\tilde{\eta}_1$ and $\tilde{\eta}_2$ are then obtained for the two responses and expressed as:

$\tilde{\eta}_1 =$ (-7.0329, -6.4527, -5.2923, -4.7121) + (0.0031, 0.0032, 0.0035, 0.0036) X_1 + (0.1440, 0.1553, 0.1777, 0.1890) X_2 + (-0.0142, -0.0037, 0.0173, 0.0279) X_3 + (-0.3623, -0.3505, -0.3265, -0.3145) X_4 (4.42)

and

$\tilde{\eta}_2 =$ (12.0353, 12.8781, 14.5639, 15.4067) + (0.0019, 0.0021, 0.0027, 0.003) X_1 + (0.2397, 0.2465, 0.2601, 0.2670) X_2 + (0.0263, 0.0454, 0.0836, 0.1027) X_3 + (-0.806, -0.7467, -0.6282, -0.5689) X_4 (4.43)

The optimal fuzzy factor levels $\tilde{X}^*_{fj}$ that optimize RB response are as follows: $\tilde{X}^*_{11}$ = (800, 800, and 800,800), $\tilde{X}^*_{21}$ = (30, 30, 30, 30), $\tilde{X}^*_{31}$ = (28.5, 28.5, 28.5, 28.5), $\tilde{X}^*_{41}$ = (6.5, 6.5, 6.5, 6.5).

Whereas the optimal fuzzy factor levels $\tilde{X}^*_{fj}$ that optimize MO response are as follows:

$\tilde{X}^*_{12}$ = (800, 800, 800, 800), $\tilde{X}^*_{22}$ = (30, 30, 30, 30), $\tilde{X}^*_{32}$ = (28.5, 28.5, 28.5, 28.5), $\tilde{X}^*_{42}$ = (6.5, 6.5, 6.5, 6.5).

By substituting the optimal factor levels for the two responses in Eqs. (4.42) and (4.43), the values of the optimal trapezoidal fuzzy responses are respectively expressed as:

$\tilde{\eta}^*_1$ = (-2.9939, -1.6174, 1.2095, 2.5888)

and

$\tilde{\eta}^*_2$ = (16.2569, 18.3935, 22.8262, 25.0428).

The trapezoidal values of pay-off matrix $\tilde{\eta}_j(x)$ for RB and MO responses are shown in Table 4.43.

Table 4.43: Pay-off matrix values for RB and MO.

	$\tilde{\eta}_1(X)$	$\tilde{\eta}_2(X)$
$\tilde{X}_1^*$	(-2.9939, -1.6174, 1.2095, 2.5888)	(16.2569, 18.3935, 22.8262, 25.0428)
$\tilde{X}_2^*$	(-2.9939, -1.6174, 1.2095, 2.5888)	(16.2569, 18.3935, 22.8262, 25.0428)

The desirability function for each response is calculated using Eq. (3.6), where $\eta_{\min} = (-12)$, $\eta_{\max} = (0)$ for RB and $\eta_{\min} = (8)$, $\eta_{\max} = (25)$ for MO are determined as an acceptable ranges. Then, the trapezoidal pay-off matrix for desirability values is shown in Table 4.44.

Table 4.44: Pay-off matrix for desirability values for RB and MO.

	$\tilde{d}_1(\tilde{\eta}_1(X))$	$\tilde{d}_2(\tilde{\eta}_2(X))$
$\tilde{X}_1^*$	(0.7505, 0.8652, 1, 1)	(0.4857, 0.6114, 0.8721, 1)
$\tilde{X}_2^*$	(0.7505, 0.8652, 1, 1)	(0.4857, 0.6114, 0.8721, 1)

The following values from Table 4.44 are determined as:

$\tilde{U}_1 = (0.7505, 0.8652, 1, 1)$, $\tilde{L}_1 = (0.7505, 0.8652, 1, 1)$

and

$\tilde{U}_2 = (0.4857, 0.6114, 0.8721, 1)$, $\tilde{L}_2 = (0.4857, 0.8721, 1)$.

The fuzzy deviation functions for RB and MO, respectively, are expressed as:

$$\tilde{D}_1(X) = 1.1604 + 2\times10^{-4}\tilde{X}_1 + 0.0226\tilde{X}_2 + 0.0212\tilde{X}_3 + 0.024\tilde{X}_4. \qquad (4.44)$$

and

$$\tilde{D}_2(X) = 1.6856 + 6\times10^{-4}\tilde{X}_1 + 0.0136\tilde{X}_2 + 0.0382\tilde{X}_3 + 0.1186\tilde{X}_4. \qquad (4.45)$$

The pay-off matrix for deviation values is constituted as shown in Table 4.45.

Table 4.45: Pay-off matrix for deviation values for RB and MO.

	$\tilde{D}_1(X)$	$\tilde{D}_2(X)$
$\tilde{X}_1^*$	(2.7586, 2.7586, 2.7586, 2.7586)	(4.4332, 4.4332, 4.4332, 4.4332)
$\tilde{X}_2^*$	(2.7586, 2.7586, 2.7586, 2.7586)	(4.4332, 4.4332, 4.4332, 4.4332)

From Table 4.45, the following values are expressed as:

$\tilde{P}_1 = (2.7586, 2.7586, 2.7586, 2.7586)$ $\tilde{Q}_1 = (2.7586, 2.7586, 2.7586, 2.7586)$

and

$\tilde{P}_2 = (4.4332, 4.4332, 4.4332, 4.4332)$ $\tilde{Q}_2 = (4.4332, 4.4332, 4.4332, 4.4332)$

The final model is constructed as

$$\begin{aligned} &Max\ \{\tilde{d}_1(X), \tilde{d}_2(X)\} \\ &Min\ \{D_1(X), D_2(X)\} \\ &s.t \end{aligned}$$

$$X = \{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4\} \in \text{[Factor Levels]}$$

By applying Zimmerman Max-Min operator, the two objectives is converted to single objective, this maximizes the minimum degree of satisfaction from the two objectives. Let $Min\ \tilde{S}_j(X) = \tilde{S} = (S^l, S^b, S^c, S^u)$, $Min\ \tilde{T}_j(X) = \tilde{T} = (T^l, T^b, T^c, T^u)$.

Then, the problem is formulated as follows:

$$\begin{aligned} &Max\ \ \tilde{S}(X) \\ &Max\ \ \tilde{T}(X) \\ &s.t \\ &\tilde{d}_j(X) - \tilde{S}(\tilde{U}_j - \tilde{L}_j) \geq \tilde{L}_j \qquad , j = 1,2. \\ &\tilde{D}_j(X) + \tilde{T}(\tilde{Q}_j - \tilde{P}_j) \leq \tilde{Q}_j \qquad , j = 1,2. \\ &X = \{\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4\} \in \text{[Factor Levels]} \end{aligned}$$

Finally, the final model using Eq. (3.13) is categorized to four types: model l, model b, model c, and model u, where models l and u are solved as in triangular membership function, whereas the remaining models as shown below:

Model b **:**

$$Max \quad 0.5\times S^{b} + 0.5\times T^{b}$$
$$s.t$$
$$0.41393 + 2.565\times10^{-4}x_1^b + 0.02x_2^b - 1.1862\times10^{-3}x_3^b - 0.03021x_4^b - 0S^b \geq 0.7505$$
$$0.2374 + 1.08824\times10^{-4}x_1^b + 0.0141x_2^b + 1.5494\times10^{-3}x_3^b - 0.0474x_4^b - 0\,S^b \geq 0.4857$$
$$1.1604 + 2\times10^{-4}x_1^b + 0.0226x_2^b + 0.0212x_3^b + 0.024x_4^b + 0\,T^b \leq 2.7586$$
$$1.6857 + 6\times10^{-4}x_1^b + 0.0136x_2^b + 0.0382x_3^b + 0.1186x_4^b + 0\,T^b \leq 4.4332$$
$$w_1 + w_2 = 1$$
$$0 \leq S^b, T^b \leq 1$$
$$X = \left\{x_1^b, x_2^b, x_3^b, x_4^b\right\} \in \text{[Factor Levels]}$$

Model c :

$$Max \quad 0.5\times S^{c} + 0.5\times T^{c}$$
$$s.t$$
$$0.51063 + 2.7992\times10^{-4}x_1^c + 0.01387x_2^c + 5.675\times10^{-4}x_3^c - 0.02821x_4^c - 0S^c \geq 0.9986$$
$$0.33653 + 1.4294\times10^{-4}x_1^c + 0.0149x_2^c + 3.7953\times10^{-3}x_3^c - 0.04044x_4^c - 0\,S^c \geq 0.7417$$
$$1.1604 + 2\times10^{-4}x_1^c + 0.0226x_2^c + 0.0212x_3^c + 0.024x_4^c + 0\,T^c \leq 2.7586$$
$$1.6857 + 6\times10^{-4}x_1^c + 0.0136x_2^c + 0.0382x_3^c + 0.1186x_4^c + 0\,T^c \leq 4.4332$$
$$w_1 + w_2 = 1$$
$$0 \leq S^c, T^c \leq 1$$
$$X = \left\{x_1^c, x_2^c, x_3^c, x_4^c\right\} \in \text{[Factor Levels]}$$

The four models on $w_1 = w_1 = 0.5$, for desirability and robustness are solved, then the optimal factor levels of models l, b,c and u are shown in Table 4.46.

Table 4.46: The optimal factor levels for biodegradation process.

	Model l	Model b	Model c	Model u
X_1^*	721.9646	780.7478	800	800
X_2^*	30	30	30	30
X_3^*	28.5	28.5	28.6816	29.2362
X_4^*	6.5	6.5	6.5	6.5

Substituting the values the of factor levels from model *l*, model *m*, and model *u*, respectively in Eq. (4.42), and in Eq. (4.43), the results are respectively expressed as:

$\tilde{\eta}_1^*$ = (-3.2358, -1.6790, 1.2126, 2.6093)

and

$\tilde{\eta}_2^*$ = (16.1086, 18.3530, 22.8414, 25.1184)

By converting the $\tilde{\eta}_1^*$ to $\tilde{y}_1^*$ using Eq. (4.34) and converting $\tilde{\eta}_2^*$ to $\tilde{y}_2^*$ using Eq. (4.35), the optimal fuzzy responses are respectively obtained:

$\tilde{y}_1^*$ = (0.6890, 0.8242, 1.1498, 1.3504)

and

$\tilde{y}_2^*$ = (0.0555, 0.0721, 0.1209, 0.1565)

Plotting the RB trapezoidal fuzzy response at optimal combination of factor levels $\tilde{y}_1^*$, is represented in Figure 4.23. Similarly, plotting the MO triangular fuzzy response at optimal combination of factor levels $\tilde{y}_2^*$, is represented in Figure 4.24.

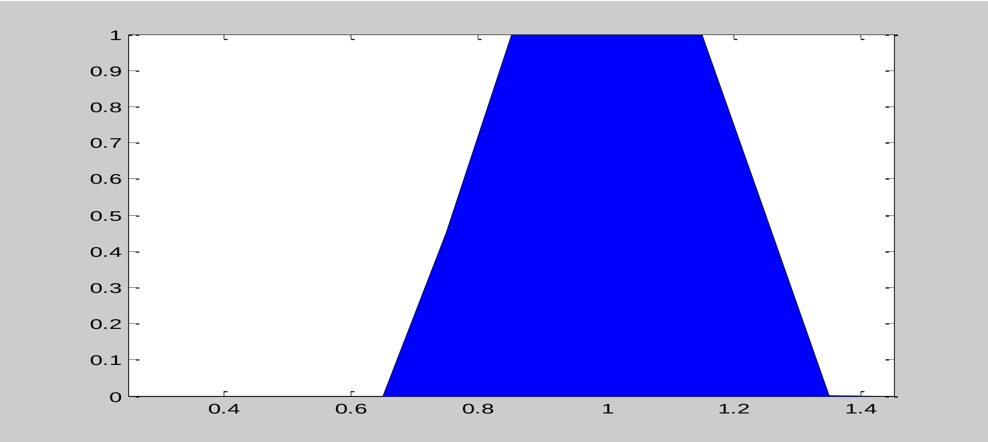


Figure 4.23: The RB trapezoidal fuzzy response at optimal combination of factor levels.

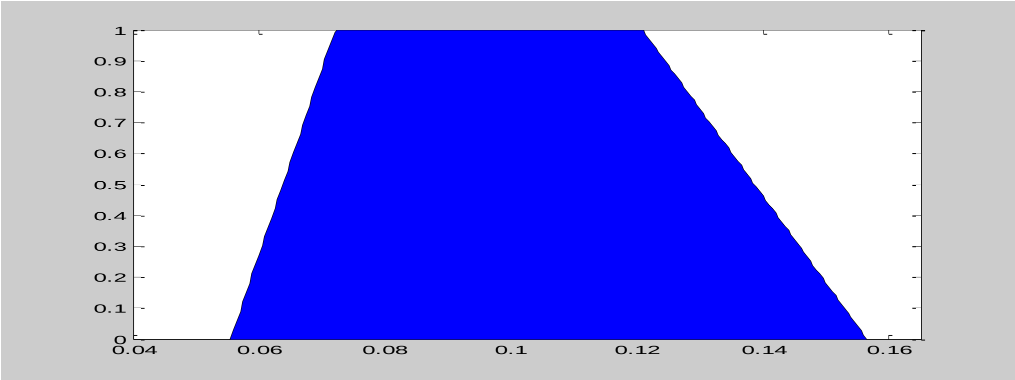


Figure 4.24: The MO trapezoidal fuzzy response at optimal combination of factor levels.

Comparing between triangular and trapezoidal membership functions, the following results are obtained:

i. The proposed approach with each of the two membership functions provides a larger satisfactory region.

ii. The triangular membership function increases the satisfactory region, and represents the ideal (mean) solution as one value; however, trapezoidal membership function increases the flexibility of the fuzzy models, and represents the ideal (mean) solution as ranges. Table 4.47 the ideal solution for both RB and MO.

Table 4.47: The ideal solutions for RB and MO responses.

Responses	Triangular Ideal Solution	Trapezoidal Ideal Solution
RB	(0.9813)	[0.8242- 1.1498]
MO	(0.0932)	[0.0721-0.1209]

iii. Both membership functions deviation and fuzziness, and all inherent variability in the collected data have been minimized and optimized, by considering each replicate, in contrast with the traditional regression approaches.

Chapter Five
Research Results

Depending on the results of the previous case studies and comparing between triangular and trapezoidal membership functions, the following results are obtained:

1. The proposed approach with each of the two membership functions provides a larger satisfactory region.
2. The triangular membership function increases the satisfactory region, and represents the ideal (mean) solution as one value; however, trapezoidal membership function increases the flexibility of the fuzzy models, and represents the ideal (mean) solution as ranges.
3. Both membership functions deviation and fuzziness, and all inherent variability in the collected data have been minimized and optimized, by considering each replicate, in contrast with the traditional regression approaches.
4. Showing factor levels as a triangular/trapezoidal fuzzy number has a capability to change them at an acceptable range based on experimental conditions.
5. Considering decision preferences for optimizing mean (desirability function) or variance (deviation function) by assigning importance weight to them in final model.
6. Setting factor levels at the optimal combination expressed anticipated improvements in the three case studies.

The above results and advantages make the proposed approach well suited for practical applications.

Chapter Six

Conclusions and Future Research

In this chapter, the main conclusions of this thesis and the future researches are provided.

6.1. Conclusions

This thesis proposes an efficient approach for optimizing the multi-response problem in the Taguchi method utilizing fuzzy regression-based approach. Optimization of multi-responses in the Taguchi method is applied to determine optimal fuzzy responses of a process in a satisfactory region and reduce variation of responses by applying fuzzy set theory. The proposed approach constitutes a regression model based on replicates of S/N ratio for each response, triangular/trapezoidal fuzzy number applied to consider mean and variance of data simultaneously and aggregate regression models so that a fuzzy regression model expresses each response, where regression model based on fuzzy data shows a very beneficial characteristic of enhanced generalization of data patterns compared to the regression models that are based on crisp data only. Solving the model, the obtained optimal factor levels are fuzzy numbers so that a larger satisfactory region could be provided. So the proposed approach can be used for optimizing the multi-response problem for a wide range in the manufacturing applications on the Taguchi method.

Two case studies in manufacturing, which were investigated in previous literature, are employed for illustration of the proposed approach, and this approach implemented in reality case study for validation, which confirm efficiency.

6.2. Future research

Future research could be conducted to optimizing the multi-response problem in the Taguchi method with correlated multi-responses utilizing fuzzy regression based approach. Further study will involve the investigation of the use of other membership functions and an optimization approach for obtaining the optimal multi-responses in the manufacturing application on the Taguchi method.

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منهج أساسه الإرتداد الضبابي لأمثلة الردود المتعددة في طريقة التاغوتشي

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ملخــــــص

طريقة التاغوتشى هو أسلوب يستخدم على نطاق واسع لتحسين عوامل المنتج / العملية لتحسين نوعية المنتجات بتكلفة منخفضة نسبيا. وعلى الرغم من هذا، هذه الطريقة ليست تقنية فعالة سوى لتحسين واحد من الردود. في الواقع ، فإن معظم المنتجات / العمليات لديها ردود متعددة التي تهم الزبون الرئيسي ، وبالتالي ، أمثلة الردود المتعددة أصبحت ذات أهمية متزايدة. هذه الرسالة تقترح منهج فعال لأمثلة متعدد الردود في طريقة التاجوشى بمنهج أساسه الإرتداد الضبابي من خلال تطبيق نظريات المجموعة الضبابية. هذا المنهج يقترح تشكل نموذج الإرتداد على أساس مضاعفات الرد ويجمع نماذج الإرتداد، لكي يظهر كل رد بنموذج إرتداد ضبابي. نموذج الارتداد الذي يتم الحصول عليه يتضمن معاملات ضبابية ثلاثية/رباعية. وتقدم دراستان سابقتان حقق فيهما مجهزتان للتوضيح. النتائج الحسابية تظهر أن المنهج المقترح يوفر أكبر منطقة مرضية. ويتم التصديق من فاعلية النظرية المقترحة بتجريبها على تجربة مصدقة للتدليل على فعالية النهج المقترح. نتيجة لذلك ، فإن النهج المقترح يمكنه توفير مساعد كبير للممارسين لأمثلة متعدد الردود في التطبيقات الصناعية على طريقة التاغوتشى ، إذا كانت مجموعة البيانات صغيرة جدا ، أو إذا كان هناك صعوبة في التحقق من أن الخطأ يتم توزيعه بشكل طبيعي، و أو إذا كان هناك غموض في العلاقة بين العوامل والردود.